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JERK RESPONSE SPECTRUM

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ABSTRACT

Jerk, also known as jolt, is the rate of change of acceleration. It is a vector quantity and its scalar magnitude is also the third derivative of position of a body or joint of a structure. Its dimension is [length/time^3]. Excessive "jerky motion" may result in an uncomfortable stay in buildings, bridges, and ride on trains or trams and it should be designed so as to reduce the influence of the jerk. This paper presents formulas and graphs for jerk response spectrum which should be taken into account among the other response spectra (displacement, velocity, acceleration) in structural engineering design for dynamic loading including earthquake impact.

1. Introduction

Acceleration response spectrum is widely used in earthquake engineering. Acceleration is the rate of change of velocity of an object or degree of freedom in a structure with respect to time.

Response spectrum is a plot of the peak value of a response quantity (displacement, velocity, acceleration, jerk or other) as a function of the natural vibration period of the system, or a related parameter such as circular frequency. Usually it is presented with several curves for different damping ratio values.

Elastic response spectrum is a function $S_e(T)$ defined as "elastic horizontal ground acceleration response spectrum". At $T = 0$, the spectral acceleration given by this spectrum equals to the design ground acceleration on type A ground multiplied by the soil factor S [6]. Sometimes this spectrum is called Elastic Design spectrum.

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Design response spectrum $S_d(T)$ includes the behaviour factor q according to [6] or response reduction factor according to US standard. At $T = 0$, the spectral acceleration given by this spectrum equals to the design ground acceleration on type A ground multiplied by the soil factor S. Sometimes this spectrum is called Inelastic Design spectrum (with specified ductility property).

Usually if it is not mentioned on the spectrum of which function it comes, it is considered that it is a spectrum of acceleration, often referred with S_a in general, in [6] with S_e for elastic and S_d for design spectrum. The expression S_D is for spectrum of displacement. The spectrum of jerk will be noted with S_j.

Jerk is a vector, and therefore it has direction and scalar magnitude, whose SI units are m/s^3 (metres per second cubed, or $m \cdot s^{-3}$). Jerk, also known as jolt, is the rate of change of acceleration. It is a vector quantity and its scalar magnitude is also the third derivative of position of a body or a joint of a structure. Excessive "jerky motion" may result in an uncomfortable stay in buildings, bridges, and ride on trains or trams and it should be designed so as to reduce the influence of jerk. There is a need for strong ground motion data and additional studies to clarify the effects of the various factors on the intensity and shapes of response spectrum curves. In developing response spectra for design application, one should place emphasis on strong ground motions recorded in the region of the site where the spectra are to be applied [3].

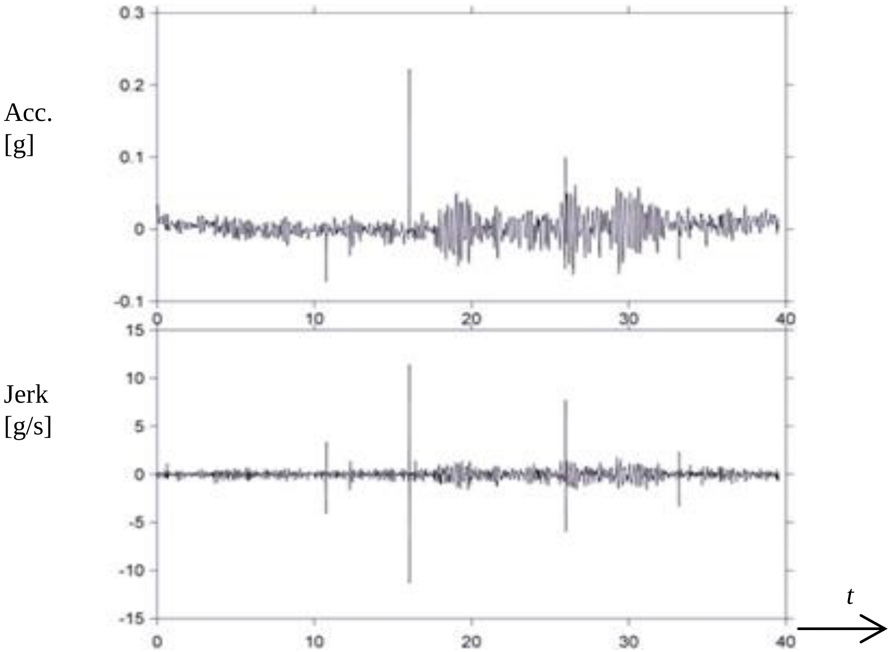

Figure 1. Horizontal component of the recordings of the 1978 Tabas earthquake in Iran [4]

The name of term "jerk" is the most common and therefore preferred for rate of change of acceleration. It is also recognised in international standards, for example in [12]. Some authors have used other names as jolt, surge, lurch, terza or TDoA (time derivative of acceleration). Jerk response spectrum is a plot of the peak or steady-state response of a jerk function of a series of oscillators.

2. Application in Common Areas

2.1. Jerk in Physics

In the physics of electromagnetism, the Abraham–Lorentz force is the recoil force on an accelerating charged particle caused by the particle emitting electromagnetic radiation. It is also called a radiation reaction force. Mathematically, that force is proportional to the jerk (the derivative of acceleration, or the third derivative of displacement).

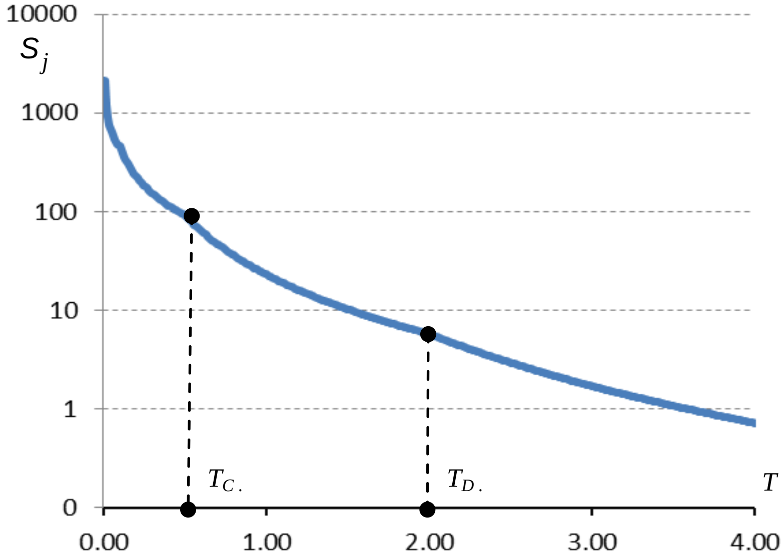

Figure 2. Elastic horizontal jerk response spectrum S_j according to (8), in log scale

2.2. Jerk in Geophysics

In geophysics, a geomagnetic jerk or secular geomagnetic variation impulse is a relatively sudden change in the second derivative of the Earth's magnetic field with respect to time [5]. These events were noted by Courtillot and others in 1978. The clearest ones, observed all over the world, happened in 1969, 1978, 1991, and 1999. It is believed that this phenomenon originates from the interior of the Earth, but their precise cause is still a matter of research. Some theories claim that geomagnetic jerk function is connected to strong earthquakes [7].

2.3. Jerk in Harmonic Vibration Theory

In [14] and [15] the third order differential equations of motion of a SDOF system in case of free and forced vibrations including the influence of third derivation of motion are considered and solved. Since jerk's influence enters in the equation of motion, to derive a solution one needs not only the initial position and velocity of the DOF, but also its initial acceleration. However, this obvious problem can be solved with a different approach. In this case, instead of the initial acceleration, additional condition is specified. This interpretation restores the coherence of the physical interpretation of the theory.

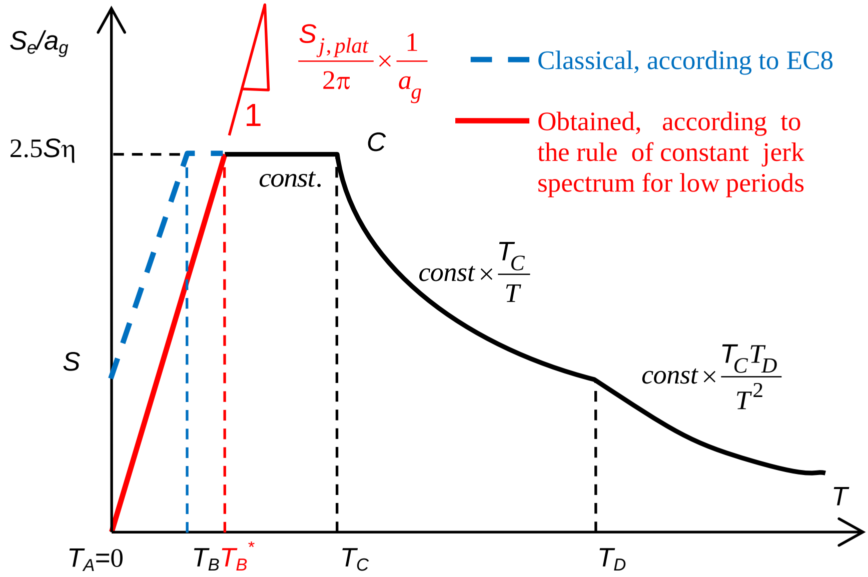

Figure 3. Elastic horizontal ground acceleration response spectra

2.4. Application in Motion of Mechanisms

Jerk is important when evaluating the destructive effect of motion on a mechanism or the discomfort caused to passengers in a vehicle. The movement of delicate instruments needs to be kept within specified limits of jerk as well as acceleration to avoid damage. When designing a train, the engineers will typically be required to keep the jerk less than 2 metres per second cubed for passengers comfort. In the aerospace industry they even have such a thing as jerkmeter – an instrument for measuring jerk [8].

2.5. Application in Geometric Design of Roads and Tracks

The principles of geometric design of roads and tracks apply to the jerk oriented orthogonally to the path of motion. Any change in curvature of the path implies non-zero jerk arising from purely geometric reasons. To avoid the unbounded (centripetal) jerk when moving from a straight path into a curve or vice versa, track transition curves are constructed, which limit the jerk by gradually increasing the curvature, to the value that belongs to the radius of the circle and the travelling speed. The theoretical optimum is achieved by the Euler spiral, which is commonly referred as clothoid. This is a curve whose curvature changes linearly with its curve length. The curvature of a circular curve is equal to the reciprocal of the radius. Using an Euler spirals in road and railroad engineering guarantees minimal constant jerk whose maximum value is 0.50 m/s^3 in some design codes for convenience purposes. A value of about 0.35 m/s^3 is recommended as a limit in railway design.

In motion control the focus is on straight linear motion, where the need is to move a system from one steady position to another (point-to-point motion). So, effectively the jerk resulting from tangential acceleration is under control. Prominent applications are elevators in people transportation, and the support of tools in machining. It is reported [11] that most passengers rate a vertical jerk of 2.00 m/s^3 in a lift ride as acceptable, 6.00 m/s^3 as intolerable. For a hospital environment 0.70 m/s^3 is suggested. In any case, limiting jerk is considered essential for riding convenience.

For passenger comfort, a train in operation will typically be required to keep jerk below 2.00 m/s^3. In the aerospace industry, a type of instrument called jerk-meter is used for measuring jerk [13].

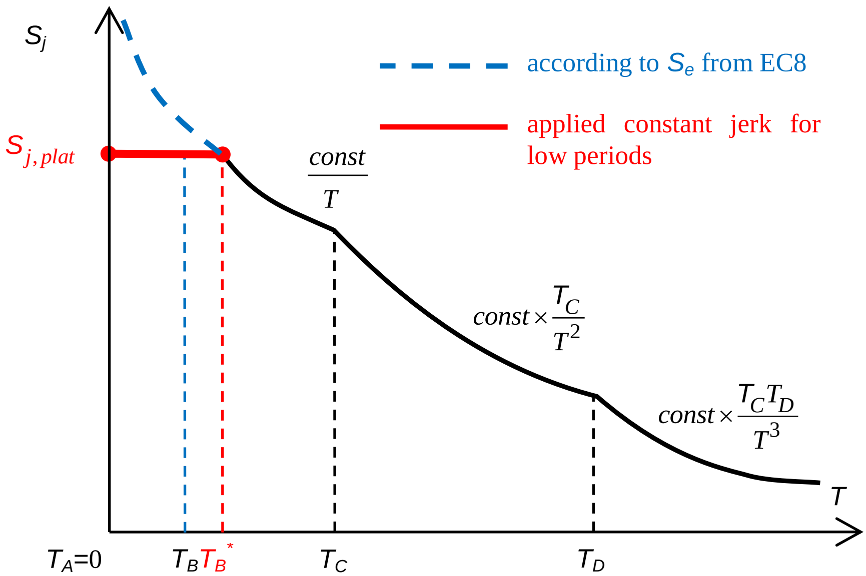

Figure 4. Elastic horizontal jerk response spectrum

2.6. Measurements for Seismic Analysis

Important characteristics of jerk are evaluated based on records from the 1999 Chi-Chi earthquake and one of its aftershocks. It is found out that the maximum of jerk at a free-field station was over 312 m/s^3, and the effective duration, between the first and the last time, 20 m/s^3, was almost one minute [10].

Although jerk has potential significance for earthquake engineering, the time-frequency characteristics of jerk are not exhaustive until now, not to mention the study on the jerk spectrum [10].

Jerk quantity can be used for analysing a time history record of earthquake. Fig. 1 shows the acceleration and jerk histories of the 1978 Tabas (Iran) earthquake. Spurious spikes are obvious in the acceleration record at 10.8 and 16 sec. The derivative of the acceleration trace (to produce the jerk) will convert a spike into a double sided pulse, making it easier to identify spikes. By doing this, spikes at 12.3, 26 and 33.2 sec are also identified in [4]. From the bottom panel of the same figure anyone can judge that the maximum jerk is about 120 m/s^3 (at 16 sec.).

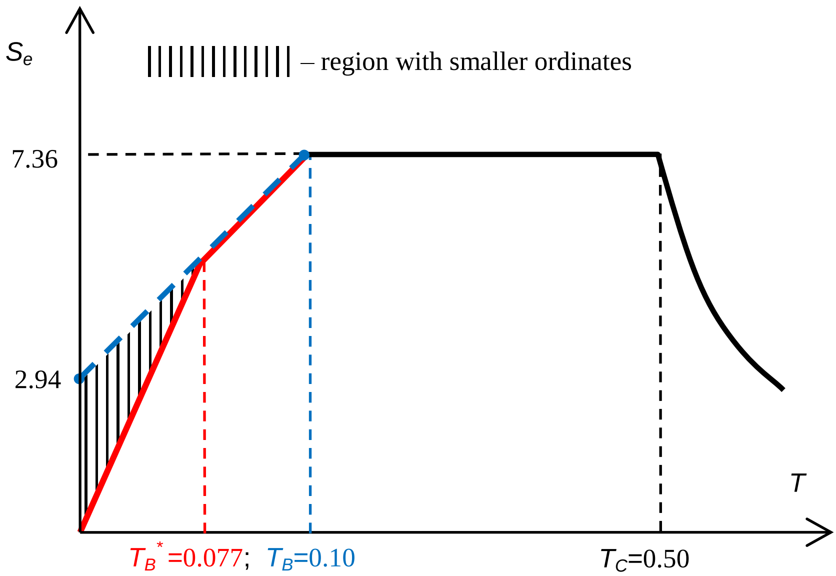

Figure 5. Modified elastic horizontal ground acceleration response spectrum

In [16] the principles and specifications (including the sensibility to noise) of a new sensor for measuring the first order derivative of acceleration are presented. Experimental results include the calibration result of the jerk sensor by using a low frequency vibration table and the earthquake wave contrast experiment.

3. Jerk Spectrum for Earthquake Analysis

3.1. According to Recurrent Formula

In earthquake engineering it is usually necessary to calculate only the so-called pseudo velocity spectral response $S_{pv}(\xi,\omega)$ defined by:

$$S_{pv}(\xi,\omega) \equiv \left[\int_0^t \ddot{u}_g(\tau)\sin\omega(t-\tau)\exp\left[-\xi\omega(t-\tau)\right]d\tau\right]_{\max}, \tag{1}$$

instead of velocity spectral response $S_v(\xi,\omega)$, whose expression is:

$$S_v(\xi,\omega) \equiv \left[\int_0^t \ddot{u}_g(\tau)\cos\omega(t-\tau)\exp\left[-\xi\omega(t-\tau)\right]d\tau\right]_{\max}. \tag{2}$$

The other basic desired spectra can be obtained by the relations [3]:

$$S_d(\xi,\omega)=\frac{1}{\omega}S_{pv}(\xi,\omega),$$
$$S_{pa}(\xi,\omega)=\omega S_{pv}(\xi,\omega). \tag{3}$$

For damping values over the range 0 ÷ 0.20, we may use the approximate relation:

$$S_a(\xi,\omega)\approx\omega S_{pv}(\xi,\omega), \tag{4}$$

which is exact for $\xi=0\%$. By using the Kelvin model, the maximum force developed in the mass is:

$$f_{s,\max}=k\,S_d(\xi,\omega)=m\,S_{pa}(\xi,\omega), \tag{5}$$

where k is the spring stiffness, m is the mass of the system. The exact expression of jerk function $j(t)$ in [10] is given by:

$$j(t)=\omega_D^2\int_0^t \ddot{u}_g(\tau)\times\begin{bmatrix}\dfrac{4\xi^3-3\xi}{\left(1-\xi^2\right)^{3/2}}\sin\omega_D(t-\tau)+\\ +\dfrac{1-4\xi^2}{1-\xi^2}\cos\omega_D(t-\tau)\end{bmatrix}\times\exp\left[-\xi\omega(t-\tau)\right]d\tau. \tag{6}$$

For practice purposes in case of low and middle damping we may use shorter expression of the (pseudo) jerk spectral response $S_j(\xi,\omega)$. The recurrence formula is:

$$S_j(\xi,\omega)\approx\omega S_a(\xi,\omega)$$
$$S_j(\xi,\omega)\approx\omega^3 S_d(\xi,\omega) \tag{8}$$

or:

$$S_j\approx\frac{2\pi}{T}S_a\approx\left(\frac{2\pi}{T}\right)^3 S_d. \tag{9}$$

With the last formula we may analyse the shape of response spectra functions in the design codes. On Fig. 2 a Jerk Response Spectrum obtained from Horizontal Elastic Response Spectrum according to [6] for ground type C, a_{gR}/g=0.25, ξ=5% is plotted. A logarithmic scale for better view is used and it is obvious that for the lowest values of natural period (less than 0.03 sec.) the jerk is greater than 1000 m/s^3 and tends to infinity. Experimental results show that the maximum value of jerk during strong earthquakes is about 600 m/s^3 [10]. An upper bound value of jerk (in the design codes) may be applied, so that the Horizontal Elastic Jerk Response Spectrum has a plateau with a value of $S_{j,plat}$. Almost every (National) design code has a similar shape of Elastic Design spectrum: with zones with constant velocity (between T_C and T_D) and constant acceleration (between T_B and T_C). Hence it follows that the zone between T_A =0 and T_B should be with a constant jerk, see Fig. 4.

By assuming an upper bound of the jerk ($S_{j,plat}$), the jerk spectrum has a shape shown on Fig. 3. Rightmost point plateau corresponds to a period of T_B^*. Corresponding function of the acceleration response spectrum (continuous line) with zero ordinate for T_A =0 is placed under the classical elastic horizontal ground acceleration response spectrum curve with zero ordinate of S for T_A =0 s., see Fig. 2. The last fact is contrary to the common practice [3] to normalize the intensity of these design spectra to the peak acceleration level:

$$\lim_{T\to 0} S_{pa}\left(\xi,T\right)=\left[\ddot{u}_g\left(t\right)\right]_{\max}. \tag{10}$$

The difference is in disregarding the jerk in the dynamic analysis. The proposed theory leads to zero value of the right-hand side of the equation (10). The slope of the line of the acceleration spectrum is $S_{j,plat}/2\pi$. The value of T_B^* can be easily found by the rule of three:

$$T_B^* = 2\pi \frac{S_{a,plat}}{S_{j,plat}}, \tag{11}$$

where

$$S_{a,plat} = a_g\ S\ \eta\ 2.5 \tag{12}$$

is according to [6]. The jerk plateau of $S_{j,plat}$ can be defined also by keeping the value of $T_B^* = T_B$. The last idea can be expressed by:

$$S_{j,plat} = 2\pi \frac{S_{a,plat}}{T_B}. \tag{13}$$

3.2. Modified Relation Between Acceleration and Jerk

As shown above, there is a contradiction between the requirement of normalization of intensity of the design spectra to the peak acceleration level (10) and the recurrence formula (8), (9). In order to keep the shape of the <u>Elastic Design spectrum</u> function $S_e(T)$ from [6] and not to specify an infinite value of corresponding jerk for low periods we need different relation between Acceleration and Jerk spectra. The proposed (modified) formula for the plateau value of the Elastic Design Jerk spectrum is:

$$\begin{aligned} S_{j,plat} &= \omega_B\left[S_e\left(T_B\right)-S_e\left(0\right)\right] \\ S_{j,plat} &= \frac{2\pi}{T_B} a_g S\left(2.5\eta-1\right) \end{aligned} \tag{14}$$

for $0 \le T \le T_B$. For $T > T_B$ the expression of jerk spectrum is according to the recurrent relationship:

$$S_j = \frac{2\pi}{T} S_a. \tag{15}$$

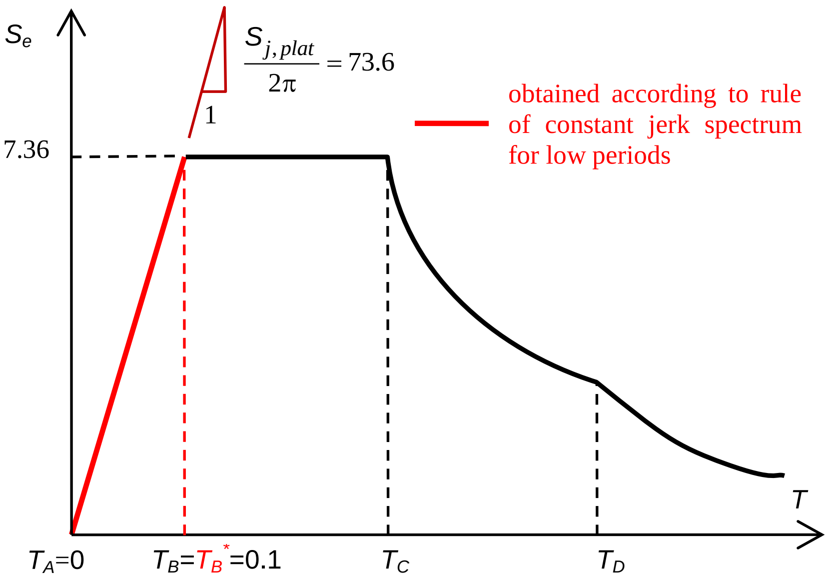

Figure 6. Modified elastic horizontal ground acceleration response spectrum in case of constant jerk for *T*≤*T*$_B$

4. Example

Build a jerk response spectrum curve corresponding to ground type C [6] (spectrum type 1 according Bulgarian National Annex with soil factor $S = 1.20$), importance factor $\gamma_I = 1$, $a_{gR} = 0.25\ g$, damping ratio $\xi = 5\%$, for three variants with the following assumptions:

– fixed upper bound value of the jerk and recurrent relationship;

– to keep the period $T_B^* \equiv T_B$ as rightmost point plateau and recurrent relationship;

– to keep the period $T_B^* \equiv T_B$ as rightmost point plateau and modified relationship.

Compare with variants based on recommended spectra types 1 and 2 in [6].

Solution: Assuming an upper bound of the jerk $S_{J,plat} = 600$ m/s^3 results in acceleration plateau $S_{a,plat} = 7.3575$ (11). With formula (10) the value of $T_B^* = 0.077$ sec. In that case the acceleration response spectrum function has an additional kink, see Fig. 5. Assuming an upper bound of the jerk $S_{j,plat} = 320$ m/s^3 as mentioned in [10] leads to $T_B^* = 0.144$ sec., which graphics is similar to Fig. 3.

For the purpose of preserving the value of the period $T_B^* \equiv T_B = 0.1$ sec., the upper bound of the jerk is $S_{j,plat} = 462.3$ m/s^3, and the whole corresponding Elastic Design spectrum should be as on the Fig. 6.

Table 1. Plateau Values of Elastic Design Acceleration and Jerk Spectra

Code (Annex), type spectrum (source)	EC8 (BG Annex), type 1 (far field)	EC8 (Recomm.), type 1 (far field)	EC8 (Recomm.), type 2 (near field)
S (soil factor)	1.20	1.15	1.50
T_B	0.10 s.	0.20 s.	0.10 s.
$S_{a,plat}$	7.36 m/s^2	7.05 m/s^2	9.20 m/s^2
$S_{j,plat}$	277.4 m/s^3	132.9 m/s^3	346.7 m/s^3

The last analysis with the modified relationship (14) is based on three variants including far field and near field source [6]. In Bulgarian Annex no type 2 is specified. Using the proposed formula (14), the source data and the plateau values of the Elastic Design Jerk spectrum are presented in Tab. 1. These results are similar to the maximum of jerk at the free-field station records from the 1999 Chi-Chi earthquake ($312\ m/s^3$, M7.6).

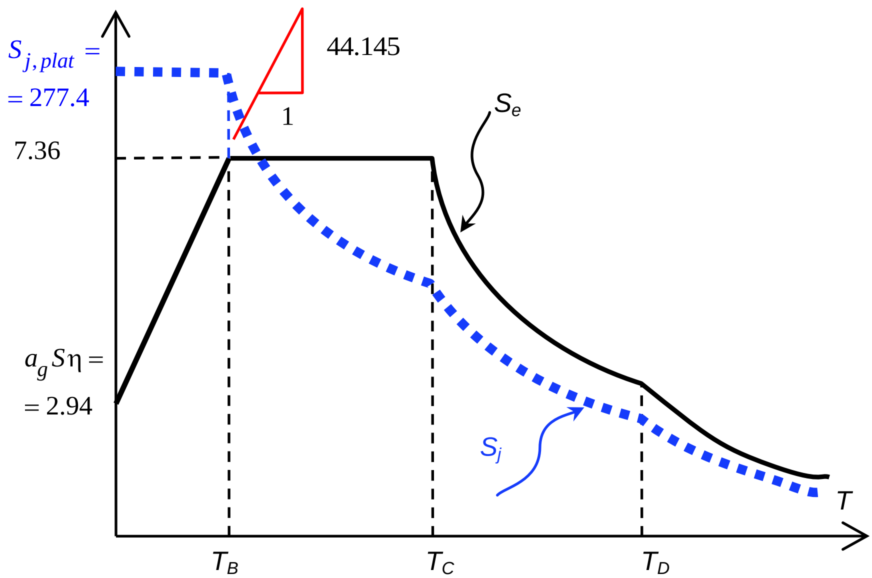

Figure 7. Acceleration and Jerk response spectra according to modified relation between them (14), with example values, based on BG Annex

5. Conclusion

Assuming that the four quantities of motion (displacement, velocity, acceleration and jerk) are reliant by their spectra on recurrent relationships leads to decreasing the elastic horizontal ground acceleration response spectrum curve in the low-period region. This will reduce the influence of the higher natural frequencies in a modal dynamic analysis.

Using an approach regarding a recurrence formula of jerk spectrum proportional to acceleration spectrum gives results in contradiction with the common practice of normalizing the intensity of these design spectra to the peak acceleration level. Additional disadvantage may appear depending on the way of assuming the connection between second and third derivative of motion: assuming a value of jerk spectrum plateau leads to additional point in the acceleration spectrum function. Applying a modified Elastic Design spectrum (with shape as on Fig. 5) will decrease the influence of higher modes in dynamic analysis. It is necessary to carry out a more thorough analysis how to include the influence of jerk on the global dynamic behavior of a structure.

Using an approach with the proposed formula (14) of jerk spectrum proportional to acceleration gives a good results comparing with ground (earthquake) motion recorded data.

As seen from Tab. 1, near field case in design codes produces (2.6 times-) greater values of jerk spectra than far field (type 1), which is a good confirmation, because the influence of the jerk diminishes with the distance.

It is necessary to carry out a more thorough analysis including establishing the influence of the change of position of the left end of the Elastic Design acceleration spectrum plateau onto the dynamic behavior of a structure. The results also show two times greater maximum jerk if the point T_B is moved from 0.2 to 0.1 sec. The last fact is expected, because the slope of the acceleration spectrum curve becomes two times steeper in that region.

Further analysis should reveal the impact of jerk response spectrum function on the MPMR (Modal Participating Mass Ratio) of the higher modes. However, in any case, it is clear that for near field source of ground motion more eigenmodes should be taken into account.

Acknowledgement

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СПЕКТЪР НА РЕАГИРАНЕ ЗА ДЖЪРКОВЕТЕ

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Ключови думи: *джърк, земетръс, спектър на реагиране, псевдо-джърк*

РЕЗЮМЕ

Джърк (jerk) се нарича степента на изменение на ускорението. Това е векторна физична величина и нейната скаларна големина е също така третата производна по време на положението на тяло или възел от конструкция. Единицата за джърк е [дължина/време^3] и може да се представи като втора производна на скоростта по време. Прекалено „джърково“ движение може да доведе до некомфортен престой в сгради, на мостове, при движение в превозни средства и при проектиране трябва да се редуцира влиянието на джърка. В тази статия са представени формули и графики за спектър на реагиране за джърковете, които трябва да бъдат взимани под внимание заедно с останалите спектри на реагиране (преместване, скорост и ускорение) при проектиране на строителни конструкции на динамични въздействия, включително земетръс.

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