ГОДИШНИК НА УНИВЕРСИТЕТА ПО АРХИТЕКТУРА, СТРОИТЕЛСТВО И ГЕОДЕЗИЯ
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DERIVATIVE VIEW FACTORS BETWEEN PARALLEL AND NON-PARALLEL RECTANGULAR SURFACES

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ABSTRACT

Estimation of the view factors between two surfaces is an important problem in radiation heat transfer with many applications in building physics. They are used to determine the energy exchanged between various building surfaces. Examples that may be cited here are emitting and absorbing walls of buildings, ceiling and floor areas, PV panels, horizontal and inclined roofs. There are different approaches to solve this problem – some of them are analytical, others – numerical. The examined surfaces can be parallel or non-parallel, that share or not share a common edge. Some basic view factors for different geometries are included in the online or printed catalogs. Other view factors can be derived from the basic VF with the help of View Factor Algebra that includes some fundamental relations between view factors. The aim of this article is to present equations for the most necessary derivative view factors between parallel and non-parallel rectangular surfaces. The source of four VBA functions is included to help in the calculations of view factors between rectangular surfaces in general arrangements.

1. Introduction

Estimation of the view factors between two surfaces is an important problem in radiation heat transfer with many applications in the determination of the energy exchanged between various building surfaces.

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There are different approaches to solve this problem – some of them are analytical, others – numerical. The examined surfaces can be parallel or non-parallel, that share or not share a common edge. Some basic view factors for different geometries are included in the online or printed catalogs [1]. Other view factors can be derived from the basic VF with the help of View Factor Algebra that includes some fundamental relations between view factors. The aim of this article is to present equations for the most necessary derivative view factors between parallel and non-parallel rectangular surfaces.

2. Analysis

2.1. Radiation exchange between any two surfaces

For any two black surfaces the thermal radiation exchange is given by Eq. (1):

$$Q_{1-2} = \sigma(T_1^4 - T_2^4)A_1F_{1-2} = \sigma(T_2^4 - T_1^4)A_2F_{2-1}\,. \tag{1}$$

Within Heat Transfer terminology the term $F_{1\text{-}2}$ is known as "configuration factor" (CF). There are also other names for the latter such as "view factor", "geometry factor", "angle factor" or "shape factor". For any two elemental surfaces such as those shown in Fig. 1, $F_{1\text{-}2}$ is given as Eq. (2):

$$F_{1-2} = \frac{1}{A_1}\int_{A_1}\int_{A_2}\frac{\cos\Phi_1\cos\Phi_2}{\pi R^2}\,dA_2\,dA_1\,, \tag{2}$$

where R is the distance between both differential elements dA_1 and dA_2; A_1 and A_2 are the faces of both surfaces; Φ_1 and Φ_2 are the angles between the normal vectors to both differential elements and the line between their centers.

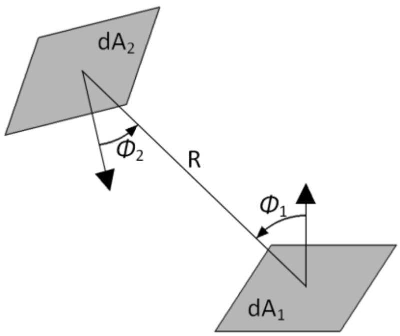

Fig. 1. Defining geometry for configuration factor [1]

In addition, to thermal radiation exchange, view factor also finds its application in the assessment of building cooling load and the design of solar thermal collector and photovoltaic systems where the amount of incident solar energy from the sun, sky and ground reflections sought. The concept of view factors assumes that emitting is uniform (isotropic).

The fundamental integral for two rectangular surfaces A_1 with dimensions $a \times b$ and A_2 with dimensions $c \times d$ is Eq. (3):

$$F_{1-2} = \frac{1}{ab}\int_{x_1=0}^{a}\int_{y_1=0}^{b}\int_{x_2=0}^{c}\int_{y_2=0}^{d}\frac{\cos\Phi_1\cos\Phi_2}{\pi R^2}dy_2dx_2dy_1dx_1 \ . \qquad (3)$$

2.1.1. Orthogonal case

One of the most revered sources of reference for configuration factor is the text of Siegel and Howell [2]. It contains a catalog of configuration factor for different geometries. The cases, which find ready application with respect to building services, are two rectangular parallel surfaces and surfaces that are perpendicular to each other.

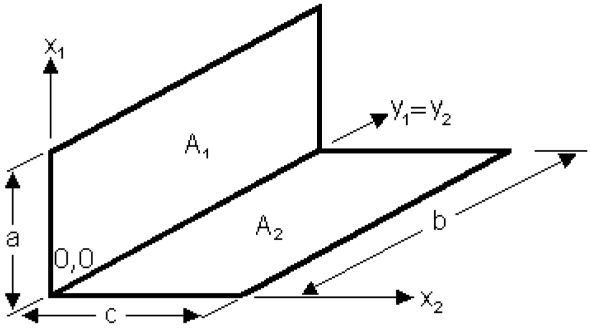

Fig. 2. Two orthogonal surfaces with one common edge

For two perpendicular rectangular surfaces with a common edge *b* (Fig. 2), the resulting integral is Eq. (4):

$$F_{1-2} = \frac{1}{ab}\int_{x_1=0}^{a}\int_{y_1=0}^{b}\int_{x_2=0}^{c}\int_{y_2=0}^{b}\frac{x_1x_2}{\pi\left[x_1^2+x_2^2+(y_1-y_2)^2\right]^2}dy_2dx_2dy_1dx_1 \ . \qquad (4)$$

The view factor – solution of this integral, is Eq. (5), where $N = c / b$ and $L = a / b$ [3]:

$$F_{1-2} = \frac{1}{\pi L}\left(\begin{array}{l} L\tan^{-1}\left(\frac{1}{L}\right)+N\tan^{-1}\left(\frac{1}{N}\right)-\sqrt{N^2+L^2}\tan^{-1}\left(\frac{1}{\sqrt{N^2+L^2}}\right) \\ +\frac{1}{4}\left\{\ln\left[\frac{(1+L^2)(1+N^2)}{1+L^2+N^2}\right]+L^2\ln\left[\frac{L^2(1+N^2+L^2)}{(1+L^2)(1+N^2)}\right]+N^2\ln\left[\frac{N^2(1+N^2+L^2)}{(1+N^2)(N^2+L^2)}\right]\right\}\end{array}\right). \qquad (5)$$

2.1.2. Tilted surface

A more generalized version of the above case is, however, the one where the two surfaces A_1 and A_2 are not perpendicular to each other, as shown in Fig. 3.

This generalized case once again has many applications such as solar energy reflected off ground and incident on a sloping roof, solar thermal water or air collectors or indeed photovoltaic modules.

The integration of Eq. (2) for the case under discussion is rather involved. It does not lead to an exact solution, as was provided for the special case of $\Phi = 90°$ – see Eq. (5). It rather leads to a partial, analytically integrable, one part, and the other part that is only numerically obtained.

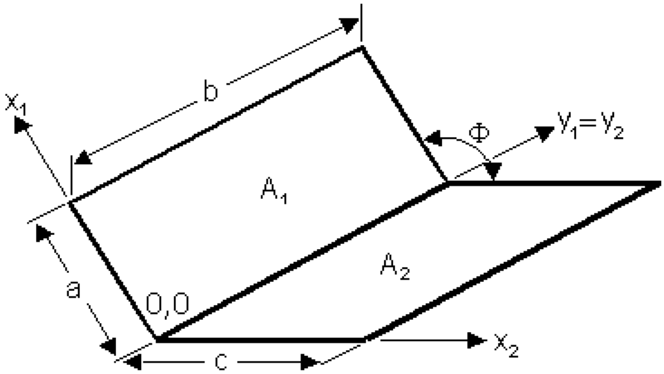

Fig. 3. Two rectangular surfaces with one common edge and included angle of Φ

If we apply Eq. (3) to two rectangular surfaces A_1 with dimensions $a \times b$ and A_2 with dimensions $c \times b$, with angle Φ between them (Fig. 3), then $\beta = \pi - \Phi$ and the resulting integral is Eq. (6):

$$F_{1-2} = \frac{1}{ab}\int_{x_1=0}^{a}\int_{y_1=0}^{b}\int_{x_2=0}^{c}\int_{y_2=0}^{b} \frac{x_1 x_2 \sin^2\beta}{\pi\left[x_1^2 + x_2^2 + 2x_1x_2\cos\beta + (y_1-y_2)^2\right]^2} dy_2 dx_2 dy_1 dx_1 \ . \qquad (6)$$

The solution of this integral is Eq. (7), where $A = c/b$, $B = a/b$, $C = A^2 + B^2 - 2AB\cos\Phi$ and $D = \sqrt{1 + A^2\sin^2\Phi}$ [4]:

$$\begin{aligned} F_{1-2} = &-\frac{\sin 2\Phi}{4\pi B}\left[AB\sin\Phi + \left(\frac{\pi}{2} - \Phi\right)\left(A^2 + B^2\right) + B^2\tan^{-1}\left(\frac{A - B\cos\Phi}{B\sin\Phi}\right) + \right. \\ &\left. + A^2\tan^{-1}\left(\frac{B - A\cos\Phi}{A\sin\Phi}\right)\right] + \frac{\sin^2\Phi}{4\pi B}\left\{\left(\frac{2}{\sin^2\Phi} - 1\right)\ln\left[\frac{(1+A^2)(1+B^2)}{1+C}\right]\right\} + \\ &+ \frac{\sin^2\Phi}{4\pi B}\left\{B^2\ln\left[\frac{B^2(1+C)}{C(1+B^2)}\right] + A^2\ln\left[\frac{A^2(1+A^2)^{\cos 2\Phi}}{C(1+C)^{\cos 2\Phi}}\right]\right\} + \\ &+ \frac{1}{\pi}\tan^{-1}\left(\frac{1}{B}\right) + \frac{A}{\pi B}\tan^{-1}\left(\frac{1}{A}\right) - \frac{\sqrt{C}}{\pi B}\tan^{-1}\left(\frac{1}{\sqrt{C}}\right) + \\ &+ \frac{\sin\Phi\sin 2\Phi}{2\pi B}AD\left[\tan^{-1}\left(\frac{A\cos\Phi}{D}\right) + \tan^{-1}\left(\frac{B - A\cos\Phi}{D}\right)\right] + \\ &+ \frac{\cos\Phi}{\pi B}\int_0^B \sqrt{1 + z^2\sin^2\Phi}\left[\tan^{-1}\left(\frac{z\cos\Phi}{\sqrt{1+z^2\sin^2\Phi}}\right) + \tan^{-1}\left(\frac{A - z\cos\Phi}{\sqrt{1+z^2\sin^2\Phi}}\right)\right]dz. \end{aligned} \qquad (7)$$

The last part of Eq. (7) is unsolvable integral. This explains why a complete analytical solution of Eq. (6) does not exist. The view factor F_{1-2} can be estimated partially analytically, partially numerically.

2.1.3. Parallel surfaces

For two parallel directly opposite rectangular surfaces (Fig. 4), Eq. (3) will have to be modified with these values of $R = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ and $\cos\Phi_1 = \cos\Phi_2 = c / R$. The resulting integral for the estimation of $VF_{1\text{-}2}$ is Eq. (8):

$$F_{1-2} = \frac{c^2}{\pi ab} \int_{x_1=0}^{a} \int_{y_1=0}^{b} \int_{x_2=0}^{a} \int_{y_2=0}^{b} \frac{1}{\left[(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2\right]^2} dy_2 dx_2 dy_1 dx_1 \ . \qquad (8)$$

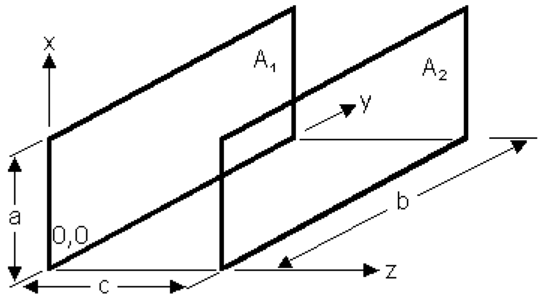

Fig. 4. Two parallel directly opposite rectangular surfaces

The configuration factor – solution of this integral, is Eq. (9) [5], where $X = a / c$ and $Y = b / c$:

$$F_{1-2} = F_{2-1} = \frac{2}{\pi XY} \left(\begin{array}{l} X\sqrt{1+Y^2}\tan^{-1}\left(\dfrac{X}{\sqrt{1+Y^2}}\right) + Y\sqrt{1+X^2}\tan^{-1}\left(\dfrac{Y}{\sqrt{1+X^2}}\right) \\ -X\tan^{-1}(X) - Y\tan^{-1}(Y) + \ln\left[\dfrac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right]^{1/2} \end{array} \right). \qquad (9)$$

2.2. View factor algebra

The view factor algebra is a combination of basic configuration factors between surfaces with different geometries and some fundamental relations between them [1]:

Superposition rules: Two superposition rules could be defined for the view factors to surfaces. They help to estimate the view factors which cannot be evaluated directly.

Rule 1: The product of the view factor F_{i-j} from a surface i to surface j and the area A_i of surface i is equal to the sum of the products of the view factors from the parts of surface i to surface j and their areas.

$$F_{i-j} A_i = \sum_{k=1}^{N} F_{i_k - j} A_{i_k} \ . \qquad (10)$$

Rule 2: The view factor F_{i-j} from a surface *i* to surface *j* is equal to the sum of the view factors from the surface *i* to the parts of the surface *j*.

$$F_{i-j} = \sum_{k=1}^{N} F_{i-j_k} \; . \tag{11}$$

Summation rule: The sum of the view factors from a given surface in an enclosure, including the possible self-view factor for concave surfaces, is 1.

Reciprocity relation: A reciprocity relation between two opposite view factors of two isotropic emitting / receiving surfaces exists and allows the calculation of a view factor from the knowledge of its reciprocal:

$$A_i F_{i-j} = A_j F_{j-i} \; . \tag{12}$$

Bounding: View factors are bounded to $0 \le F_{i-j} \le 1$ by definition.

3. Analysis of configurations

The aim of this article is to present equations for the most necessary derivative view factors between parallel and non-parallel rectangular surfaces. Thus we will study different configurations of two rectangles, which are parallel or non-parallel.

3.1. Configuration 1

Let us have two rectangular surfaces *i* and *j* with a common edge and let each of them have three rectangular parts: $A_{1,2,3} = A_1 + A_2 + A_3$ and $A_{1',2',3'} = A_{1'} + A_{2'} + A_{3'}$ (Fig. 5). Let us apply VFA to estimate view factors between the parts of both surfaces – separately or in combinations.

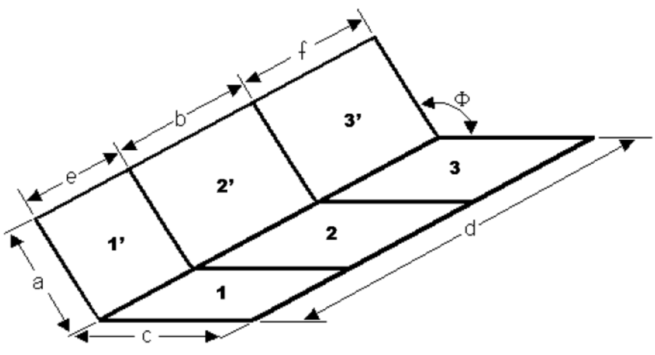

Fig. 5. Configuration 1 – two rectangular surfaces, subdivided into 3 parts, with one common edge

If we apply the Eq. (3) to the surfaces in this configuration, we could represent view factors $F_{2-1'}$ and $F_{1-2'}$ as quadruple integral in Eqs. (13) and (14).

$$F_{2-1'} = \frac{1}{bc}\int_{x_2=0}^{c}\int_{y_2=0}^{b}\int_{x_1=0}^{a}\int_{y_1=-e}^{0}\frac{\cos\theta_i.\cos\theta_j}{\pi R^2}dy_1dx_1dy_2dx_2\ , \tag{13}$$

$$F_{1-2'} = \frac{1}{ec}\int_{x_2=0}^{c}\int_{y_1=-e}^{0}\int_{x_1=0}^{a}\int_{y_2=0}^{b}\frac{\cos\theta_i.\cos\theta_j}{\pi R^2}dy_2dx_1dy_1dx_2\ . \tag{14}$$

If we compare last two Eqs. (13) and (14) we could see the relationship between these view factors – Eq. (15):

$$b.F_{2-1'} = e.F_{1-2'}\ . \tag{15}$$

This relationship, added to the other relationships between the view factors, can help us to compute derivative view factors like $F_{1,2-1'}$, etc.

$$F_{1,2-1'} = \frac{1}{2}\left(F_{1,2-1',2'} + \frac{e}{e+b}F_{1-1'} - \frac{b}{e+b}F_{2-2'}\right). \tag{16}$$

The view factors $F_{1,2-1',2'}$, $F_{1-1'}$ and $F_{2-2'}$ are basic view factors, they can be estimated with Eq. (7). The equations of the other derivative factors are estimated with the help of *K* terms, which are defined by $K_{m-n} = A_mF_{m-n}$. The results, expressed in *K* term, are arranged in Appendix Table A1. On their base the equations of derivative VF, presented in Appendix Table A2, are prepared.

3.2. Configuration 2 – generalized case for non-parallel rectangles

Let us have two rectangular surfaces with a common edge, separated by given angle ϕ, and let each of them have six rectangular parts: $A_{123456} = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$ and $A_{1'2'3'4'5'6'} = A_{1'} + A_{2'} + A_{3'} + A_{4'} + A_{5'} + A_{6'}$ (Fig. 6). We applied the resulting equations from configuration 1 and view factor algebra and proved in [6] the Eq. (17) for the estimation of derivative view factor $F_{1-3'}$ for an inclined receiving surface. There term $K_{(m)^2} = A_mF_{m-m'}$.

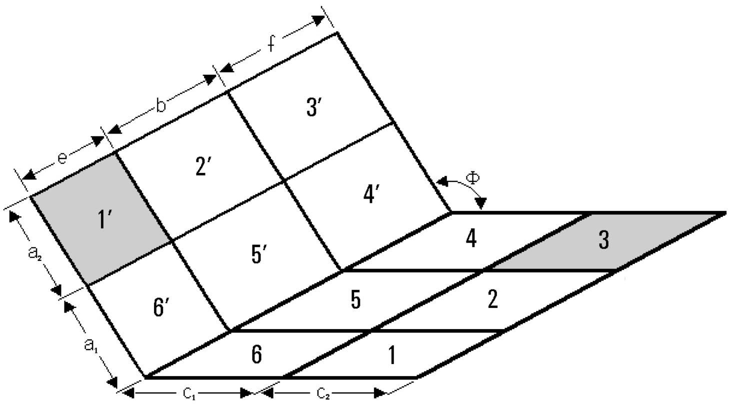

Fig. 6. Configuration 2 – generalized inclined-rectangle arrangement. The *VF* of part 1 of surface A_{123456} to part 3’ of surface $A_{1'2'3'4'5'6'}$ can be estimated with the help of view factor algebra

$$A_1F_{1-3'} = \frac{1}{2}\begin{pmatrix} K_{(123456)^2} - K_{(1256)^2} - K_{(2345)^2} + K_{(25)^2} - K_{(4,5,6)-(1'2'3'4'5'6')} \\ +K_{(56)-(1'2'5'6')} + K_{(45)-(2'3'4'5')} - K_{5-(2'5')} - K_{(123456)-(4'5'6')} \\ +K_{(1256)-(5'6')} + K_{(2345)-(4'5')} - K_{(25)-5'} + K_{(4,5,6)^2} \\ -K_{(56)^2} - K_{(45)^2} + K_{(5)^2} \end{pmatrix}. \quad (17)$$

This equation is presented in [7] for two perpendicular surfaces.

3.3. Configuration 3 – generalized case for parallel rectangles

Let us have two directly opposite rectangular surfaces and each of them has 9 rectangular parts in three rows with the same size for both surfaces: $A_1 = A_{1'}$, $A_2 = A_{2'}$ etc. (Fig. 7).

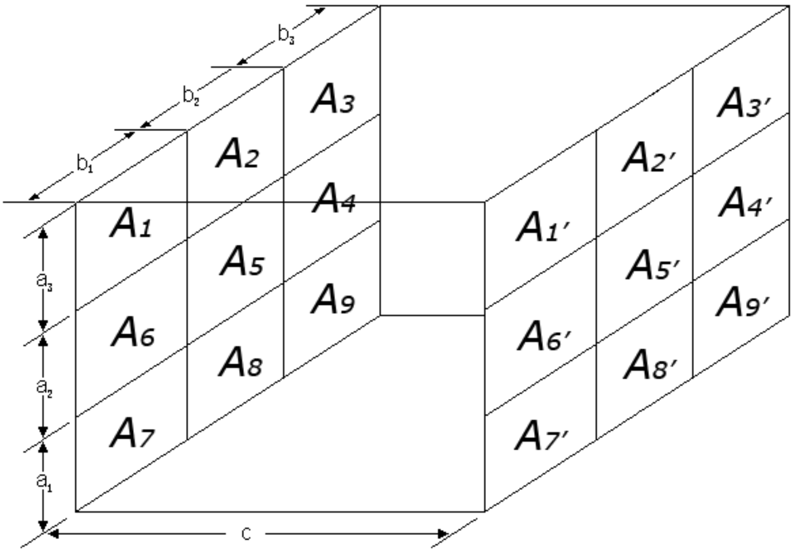

Fig. 7. Configuration 3 – two parallel surfaces subdivided horizontally and vertically into 9 parts

The final result for the view factor $F_{3'\text{-}7}$ is as follows in Eq. (18) [8], where all participating variables are basic view factors:

$$F_{3'\text{-}7} = \frac{1}{A_{3'}}\begin{pmatrix} K_{(1,2,3,4,5,6,7,8,9)^2} - K_{(2,3,4,5,8,9)^2} - K_{(1,2,5,6,7,8)^2} + K_{(2,5,8)^2} - K_{(4,5,6,7,8,9)^2} \\ +K_{(4,5,8,9)^2} + K_{(5,6,7,8)^2} - K_{(5,8)^2} - K_{(1,2,3,4,5,6)^2} + K_{(2,3,4,5)^2} + K_{(1,2,5,6)^2} \\ -K_{(2,5)^2} + K_{(4,5,6)^2} - K_{(4,5)^2} - K_{(5,6)^2} + K_{(5)^2} \end{pmatrix} / 2 \,. \quad (18)$$

This result is given in this final form in [7] also and can be useful for uniform emitting and reflecting surfaces.

4. VBA source code

The Eqs. (7), (17) and (18) were used to develop VBA source code to estimate view factors in any configuration of parallel or non-parallel rectangular surfaces. The contents of the VBA functions are listed below.

4.1. VBA source code for estimation of VF for non-parallel rectangles in generalized arrangement

Two VBA functions for estimation of VF for non-parallel rectangles in a generalized arrangement are presented below. The first of them (Function VF_Incl_Rect_To_Hor_Rect) estimates the view factor from a rectangle to an opposite rectangle with an included angle of Φ between them (Fig. 3), using Eq. (7). The next function (Function VFactor13) calculates the view factor between rectangular surfaces in a generalized arrangement, as described in section 3.2 (Fig. 6), using Eq. (17).

For arguments $a1=0$ and $c1=0$ the view factors, as described in section 3.1 (Fig. 5), could be calculated.

```
' View Factor from a rectangle to an opposite rectangle with included angle of Φ between them
Function VF_Incl_Rect_To_Hor_Rect(bb As Double, cc As Double, _
                                   aa As Double, Fi As Double) As Double
Dim a As Double, b As Double, c As Double, d As Double, VF As Double, Pi As Double
Dim Sum As Double, i As Integer, Nz As Integer, stepB As Double, z As Double
Dim CosFi As Double, SinFi As Double, Sin2Fi As Double, t As Double
    If bb <> 0 And aa <> 0 And cc <> 0 Then
        Pi = Application.Pi()
        CosFi = Cos(Fi): SinFi = Sin(Fi): Sin2Fi = Sin(2 * Fi)
        a = cc / bb: b = aa / bb
        c = a ^ 2 + b ^ 2 - 2 * a * b * CosFi: d = Sqr(1 + a ^ 2 * SinFi ^ 2)
        VF = -Sin2Fi / (4 * Pi * b) * _
            (a * b * SinFi + (Pi / 2 - Fi) * (a ^ 2 + b ^ 2) + b ^ 2 * Atn((a - b * CosFi)  _
            / (b * SinFi)) + a ^ 2 * Atn((b - a * CosFi) / (a * SinFi)))
        VF = VF + SinFi ^ 2 / (4 * Pi * b) * _
            ((2 / SinFi ^ 2 - 1) * Log((1 + a ^ 2) * (1 + b ^ 2) / (1 + c)) + _
            b ^ 2 * Log(b ^ 2 * (1 + c) / c / (1 + b ^ 2)) + a ^ 2 * Log(a ^ 2 * (1 + a ^ 2) ^ Cos(2 * Fi) _
            / c / (1 + c) ^ Cos(2 * Fi)))
        VF = VF + 1 / Pi * Atn(1 / b) + a / (Pi * b) * Atn(1 / a) - Sqr(c) * Atn(1 / Sqr(c)) / (Pi * b)
        VF = VF + SinFi * Sin2Fi / (2 * Pi * b) * a * d * (Atn(a * CosFi / d) + Atn((b - a * CosFi) / d))

        Sum = 0: Nz = 1000: stepB = b / Nz
        For i = 1 To Nz  'unsolvable integral
            z = (i - 0.5) * stepB
            t = Sqr(1 + z ^ 2 * SinFi ^ 2)
            Sum = Sum + t * (Atn(z * CosFi / t) + Atn((a - z * CosFi) / t))
        Next
        Sum = Sum * CosFi * stepB / (Pi * b)
        VF_Incl_Rect_To_Hor_Rect = VF + Sum
    Else
        VF_Incl_Rect_To_Hor_Rect = 0
    End If
End Function
```

```
Function VFactor13(a1 As Double, a2 As Double, c1 As Double, c2 As Double, _
                    e As Double, b As Double, f As Double, Fi As Double) As Double
Dim VF As Double
    VF = 0
    VF = VF + (e + b + f) * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(e + b + f, c1 + c2, a1 + a2, Fi)
    VF = VF + b * a1 * VF_Incl_Rect_To_Hor_Rect(b, c1, a1, Fi)
    VF = VF + b * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(b, c1 + c2, a1 + a2, Fi)
    VF = VF - (b + f) * a1 * VF_Incl_Rect_To_Hor_Rect(b + f, c1, a1, Fi)
    VF = VF - (b + f) * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(b + f, c1 + c2, a1 + a2, Fi)
    VF = VF - b * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(b, c1, a1 + a2, Fi)
    VF = VF - b * a1 * VF_Incl_Rect_To_Hor_Rect(b, c1 + c2, a1, Fi)
    VF = VF + (b + f) * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(b + f, c1, a1 + a2, Fi)
    VF = VF + (b + f) * a1 * VF_Incl_Rect_To_Hor_Rect(b + f, c1 + c2, a1, Fi)
    VF = VF - (e + b + f) * a1 * VF_Incl_Rect_To_Hor_Rect(e + b + f, c1 + c2, a1, Fi)
    VF = VF - (e + b + f) * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(e + b + f, c1, a1 + a2, Fi)
    VF = VF - (e + b) * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(e + b, c1 + c2, a1 + a2, Fi)
    VF = VF + (e + b) * a1 * VF_Incl_Rect_To_Hor_Rect(e + b, c1 + c2, a1, Fi)
    VF = VF + (e + b) * (a1 + a2) * VF_Incl_Rect_To_Hor_Rect(e + b, c1, a1 + a2, Fi)
    VF = VF + (e + b + f) * a1 * VF_Incl_Rect_To_Hor_Rect(e + b + f, c1, a1, Fi)
    VF = VF - (e + b) * a1 * VF_Incl_Rect_To_Hor_Rect(e + b, c1, a1, Fi)
    VFactor13 = VF / (2 * a2 * e)
End Function
```

4.2. VBA source code for estimation of VF for parallel rectangles in generalized arrangement

Other two VBA functions are presented below. The first of them (Function VF_Vert_Rect) estimates the view factor from a rectangle to a directly opposite parallel rectangles at distance *c* between them (Fig. 4), using Eq. (9). The next function (VFactor_v13) calculates the view factor between two parallel rectangular surfaces in generalized arrangement, as described in section 3.3 (Fig. 7), using Eq. (18).

```
' View Factor from a Vertical Rectangle to Opposite Vertical Rectangle
Function VF_Vert_Rect(a As Double, c As Double, d As Double)
' c is distance between both rectangles
    If a <> 0 And c <> 0 And d <> 0 Then
        VF_Vert_Rect = 2 / (a * d * Application.Pi()) * (a * Sqr(c ^ 2 + d ^ 2) * _
            Atn(a / Sqr(c ^ 2 + d ^ 2)) + d * Sqr(a ^ 2 + c ^ 2) * Atn(d / Sqr(a ^ 2 + c ^ 2)) _
            - a * c * Atn(a / c) - c * d * Atn(d / c) + c ^ 2 / 2 * Log((a ^ 2 + c ^ 2) * _
            (c ^ 2 + d ^ 2) / c ^ 2 / (a ^ 2 + c ^ 2 + d ^ 2)))
    Else
        VF_Vert_Rect = 0
    End If
End Function

Function VFactor_v13(a1 As Double, a2 As Double, a3 As Double, _
                      b1 As Double, b2 As Double, b3 As Double, c As Double) As Double
Dim VF As Double
    VF = 0
    VF = VF + (a1 + a2 + a3) * (b1 + b2 + b3) * VF_Vert_Rect(a1 + a2 + a3, c, b1 + b2 + b3)
    VF = VF - (a1 + a2 + a3) * (b2 + b3) * VF_Vert_Rect(a1 + a2 + a3, c, b2 + b3)
    VF = VF - (a1 + a2 + a3) * (b1 + b2) * VF_Vert_Rect(a1 + a2 + a3, c, b1 + b2)
    VF = VF + (a1 + a2 + a3) * b2 * VF_Vert_Rect(a1 + a2 + a3, c, b2)
```

```
    VF = VF - (a1 + a2) * (b1 + b2 + b3) * VF_Vert_Rect(a1 + a2, c, b1 + b2 + b3)
    VF = VF + (a1 + a2) * (b2 + b3) * VF_Vert_Rect(a1 + a2, c, b2 + b3)
    VF = VF + (a1 + a2) * (b1 + b2) * VF_Vert_Rect(a1 + a2, c, b1 + b2)
    VF = VF - (a1 + a2) * (b2) * VF_Vert_Rect(a1 + a2, c, b2)
    VF = VF - (a2 + a3) * (b1 + b2 + b3) * VF_Vert_Rect(a2 + a3, c, b1 + b2 + b3)
    VF = VF + (a2 + a3) * (b2 + b3) * VF_Vert_Rect(a2 + a3, c, b2 + b3)
    VF = VF + (a2 + a3) * (b1 + b2) * VF_Vert_Rect(a2 + a3, c, b1 + b2)
    VF = VF - (a2 + a3) * (b2) * VF_Vert_Rect(a2 + a3, c, b2)
    VF = VF + (a2) * (b1 + b2 + b3) * VF_Vert_Rect(a2, c, b1 + b2 + b3)
    VF = VF - (a2) * (b2 + b3) * VF_Vert_Rect(a2, c, b2 + b3)
    VF = VF - (a2) * (b1 + b2) * VF_Vert_Rect(a2, c, b1 + b2)
    VF = VF + (a2) * (b2) * VF_Vert_Rect(a2, c, b2)
    VFactor_v13 = VF / (4 * a1 * b1)
End Function
```

5. Conclusions

This article presents the equations for the most necessary derivative view factors between parallel and non-parallel rectangular surfaces. They are based on the basic view factors between parallel or non-parallel rectangles and on the superposition rules and reciprocity relation that are parts of View Factor Algebra. The source of four VBA functions is included to help in the calculations of view factors between rectangular surfaces in general arrangements. These view factors can be used to estimate exchanged energy between the emitting and absorbing walls of buildings, ceiling and floor areas, PV panels, horizontal and inclined roofs.

REFERENCES

1. *Howell, J. R.* A catalog of radiation heat transfer – configuration factors. Introduction. Available on: http://www.thermalradiation.net/intro.html.

2. *Siegel, R., Howell, J.* Thermal Radiation and Heat Transfer, 4th ed. New York: Taylor & Francis, 2002.

3. *Howell, J. R.* A catalog of radiation heat transfer – configuration factors. C-14: Two finite rectangles of same length, having one common edge, and at an angle of 90° to each other. Available on: http://www.thermalradiation.net/sectionc/C-14.html.

4. *Howell, J. R.* A catalog of radiation heat transfer – configuration factors. C-16: Two rectangles with one common edge and included angle of Φ. Available on: http://www.thermalradiation.net/sectionc/C-16.html.

5. *Howell, J. R.* A catalog of radiation heat transfer – configuration factors. C-11: Identical, parallel, directly opposed rectangles. Available on: http://www.thermalradiation.net/sectionc/C-11.html.

6. *Muneer, T., Ivanova, S., Kotak, Y. and Gul, M.* Finite-element view-factor computations for radiant energy exchanges. // Journal of Renewable and Sustainable Energy, Volume 7, Issue 3, May-June 2015, pp. 033108-1 – 033108-20.

7. *Holman, J. P.* Heat Transfer, 7^{th} edition. McGraw-Hill, New York, 1992.

8. *Ivanova, S., Muneer, T.* Finite-element heat-transfer computations for parallel surfaces with uniform or non-uniform emitting. // Journal of Renewable and Sustainable Energy, Volume 8, Issue 1, January-February 2016, pp. 015102-1 – 015102-16.

APPENDIX

Table A1. Equations of derivative view factors in *K* term

Derivative view factors	Equation
$F_{1\text{-}1'}$, $F_{2\text{-}2'}$, $F_{3\text{-}3'}$, $F_{1,2\text{-}1',2'}$, $F_{2,3\text{-}2',3'}$, $F_{1,2,3\text{-}1',2',3'}$ and the opposite $F_{1'\text{-}1}$, $F_{2'\text{-}2}$, $F_{3'\text{-}3}$, $F_{1',2'\text{-}1,2}$, $F_{2',3'\text{-}2,3}$, $F_{1',2',3'\text{-}1,2,3}$ are basic view factors, estimated with Eq. (7).	
$K_{1-2'} = \frac{1}{2}\left(K_{1,2-1',2'} - K_{1-1'} - K_{2-2'}\right)$	(A1)
$K_{1-3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{2,3-2',3'} - K_{1,2-1',2'} + K_{2-2'}\right)$	(A2)
$K_{2-1'} = \frac{1}{2}\left(K_{1,2-1',2'} - K_{1-1'} - K_{2-2'}\right)$	(A3)
$K_{2-3'} = \frac{1}{2}\left(K_{2,3-2',3'} - K_{3-3'} - K_{2-2'}\right)$	(A4)
$K_{3-1'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{2,3-2',3'} - K_{1,2-1',2'} + K_{2-2'}\right)$	(A5)
$K_{3-2'} = \frac{1}{2}\left(K_{2,3-2',3'} - K_{3-3'} - K_{2-2'}\right)$	(A6)
$K_{1-1',2'} = \frac{1}{2}\left(K_{1,2-1',2'} + K_{1-1'} - K_{2-2'}\right)$	(A7)
$K_{1-2',3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{2,3-2',3'} - K_{1-1'}\right)$	(A8)
$K_{2-1',2'} = \frac{1}{2}\left(K_{1,2-1',2'} - K_{1-1'} + K_{2-2'}\right)$	(A9)
$K_{2-2',3'} = \frac{1}{2}\left(K_{2,3-2',3'} - K_{3-3'} + K_{2-2'}\right)$	(A10)
$K_{3-1',2'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{1,2-1',2'} - K_{3-3'}\right)$	(A11)
$K_{3-2',3'} = \frac{1}{2}\left(K_{2,3-2',3'} + K_{3-3'} - K_{2-2'}\right)$	(A12)
$K_{1-1',2',3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{2,3-2',3'} + K_{1-1'}\right)$	(A13)

$K_{2-1',2',3'} = \frac{1}{2}\left(K_{1,2-1',2'} + K_{2,3-2',3'} - K_{1-1'} - K_{3-3'}\right)$	(A14)
$K_{3-1',2',3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{1,2-1',2'} + K_{3-3'}\right)$	(A15)
$K_{1,2-1'} = \frac{1}{2}\left(K_{1,2-1',2'} + K_{1-1'} - K_{2-2'}\right)$	(A16)
$K_{1,2-2'} = \frac{1}{2}\left(K_{1,2-1',2'} - K_{1-1'} + K_{2-2'}\right)$	(A17)
$K_{1,2-3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{1,2-1',2'} - K_{3-3'}\right)$	(A18)
$K_{2,3-1'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{2,3-2',3'} - K_{1-1'}\right)$	(A19)
$K_{2,3-2'} = \frac{1}{2}\left(K_{2,3-2',3'} - K_{3-3'} + K_{2-2'}\right)$	(A20)
$K_{2,3-3'} = \frac{1}{2}\left(K_{2,3-2',3'} + K_{3-3'} - K_{2-2'}\right)$	(A21)
$K_{1,2-2',3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{1-1'} + K_{2-2'} - K_{3-3'}\right)$	(A22)
$K_{2,3-1',2'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{1-1'} + K_{2-2'} - K_{3-3'}\right)$	(A23)
$K_{1,2-1',2',3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} + K_{1,2-1',2'} - K_{3-3'}\right)$	(A24)
$K_{2,3-1',2',3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} + K_{2,3-2',3'} - K_{1-1'}\right)$	(A25)
$K_{1,2,3-1'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{2,3-2',3'} + K_{1-1'}\right)$	(A26)
$K_{1,2,3-2'} = \frac{1}{2}\left(K_{1,2-1',2'} + K_{2,3-2',3'} - K_{1-1'} - K_{3-3'}\right)$	(A27)
$K_{1,2,3-3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} - K_{1,2-1',2'} + K_{3-3'}\right)$	(A28)
$K_{1,2,3-1',2'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} + K_{1,2-1',2'} - K_{3-3'}\right)$	(A29)
$K_{1,2,3-2',3'} = \frac{1}{2}\left(K_{1,2,3-1',2',3'} + K_{2,3-2',3'} - K_{1-1'}\right)$	(A30)

Table A2. Equations of derivative view factors

Derivative view factors	Equation
$F_{1\text{-}1'}$, $F_{2\text{-}2'}$, $F_{3\text{-}3'}$, $F_{1,2\text{-}1',2'}$, $F_{2,3\text{-}2',3'}$, $F_{1,2,3\text{-}1',2',3'}$ and the opposite $F_{1'\text{-}1}$, $F_{2'\text{-}2}$, $F_{3'\text{-}3}$, $F_{1',2'\text{-}1,2}$, $F_{2',3'\text{-}2,3}$, $F_{1',2',3'\text{-}1,2,3}$ are basic view factors, estimated with Eq. (7).	
$F_{1-2'}=\frac{1}{2}\left(\frac{e+b}{e}F_{1,2-1',2'}-F_{1-1'}-\frac{b}{e}F_{2-2'}\right)$	(A31)
$F_{1-3'}=\frac{1}{2}\left(\frac{d}{e}F_{1,2,3-1',2',3'}-\frac{b+f}{e}F_{2,3-2',3'}-\frac{e+b}{e}F_{1,2-1',2'}+\frac{b}{e}F_{2-2'}\right)$	(A32)
$F_{2-1'}=\frac{1}{2}\left(\frac{e+b}{b}F_{1,2-1',2'}-\frac{e}{b}F_{1-1'}-F_{2-2'}\right)$	(A33)
$F_{2-3'}=\frac{1}{2}\left(\frac{b+f}{b}F_{2,3-2',3'}-\frac{f}{b}F_{3-3'}-F_{2-2'}\right)$	(A34)
$F_{3-1'}=\frac{1}{2}\left(\frac{d}{f}F_{1,2,3-1',2',3'}-\frac{b+f}{f}F_{2,3-2',3'}-\frac{e+b}{f}F_{1,2-1',2'}+\frac{b}{f}F_{2-2'}\right)$	(A35)
$F_{3-2'}=\frac{1}{2}\left(\frac{b+f}{f}F_{2,3-2',3'}-F_{3-3'}-\frac{b}{f}F_{2-2'}\right)$	(A36)
$F_{1-1',2'}=\frac{1}{2}\left(\frac{e+b}{e}F_{1,2-1',2'}+F_{1-1'}-\frac{b}{e}F_{2-2'}\right)$	(A37)
$F_{1-2',3'}=\frac{1}{2}\left(\frac{d}{e}F_{1,2,3-1',2',3'}-\frac{b+f}{e}F_{2,3-2',3'}-F_{1-1'}\right)$	(A38)
$F_{2-1',2'}=\frac{1}{2}\left(\frac{e+b}{b}F_{1,2-1',2'}-\frac{e}{b}F_{1-1'}+F_{2-2'}\right)$	(A39)
$F_{2-2',3'}=\frac{1}{2}\left(\frac{b+f}{b}F_{2,3-2',3'}-\frac{f}{b}F_{3-3'}+F_{2-2'}\right)$	(A40)
$F_{3-1',2'}=\frac{1}{2}\left(\frac{d}{f}F_{1,2,3-1',2',3'}-\frac{e+b}{f}F_{1,2-1',2'}-F_{3-3'}\right)$	(A41)
$F_{3-2',3'}=\frac{1}{2}\left(\frac{b+f}{f}F_{2,3-2',3'}+F_{3-3'}-\frac{b}{f}F_{2-2'}\right)$	(A42)
$F_{1-1',2',3'}=\frac{1}{2}\left(\frac{d}{e}F_{1,2,3-1',2',3'}-\frac{b+f}{e}F_{2,3-2',3'}+F_{1-1'}\right)$	(A43)
$F_{2-1',2',3'}=\frac{1}{2}\left(\frac{e+b}{b}F_{1,2-1',2'}+\frac{b+f}{b}F_{2,3-2',3'}-\frac{e}{b}F_{1-1'}-\frac{f}{b}F_{3-3'}\right)$	(A44)

$F_{3-1',2',3'}=\frac{1}{2}\left(\frac{d}{f}F_{1,2,3-1',2',3'}-\frac{e+b}{f}F_{1,2-1',2'}+F_{3-3'}\right)$	(A45)
$F_{1,2-1'}=\frac{1}{2}\left(F_{1,2-1',2'}+\frac{e}{e+b}F_{1-1'}-\frac{b}{e+b}F_{2-2'}\right)$	(A46)
$F_{1,2-2'}=\frac{1}{2}\left(F_{1,2-1',2'}-\frac{e}{e+b}F_{1-1'}+\frac{b}{e+b}F_{2-2'}\right)$	(A47)
$F_{1,2-3'}=\frac{1}{2}\left(\frac{d}{e+b}F_{1,2,3-1',2',3'}-F_{1,2-1',2'}-\frac{f}{e+b}F_{3-3'}\right)$	(A48)
$F_{2,3-1'}=\frac{1}{2}\left(\frac{d}{b+f}F_{1,2,3-1',2',3'}-F_{2,3-2',3'}-\frac{e}{b+f}F_{1-1'}\right)$	(A49)
$F_{2,3-2'}=\frac{1}{2}\left(F_{2,3-2',3'}-\frac{f}{b+f}F_{3-3'}+\frac{b}{b+f}F_{2-2'}\right)$	(A50)
$F_{2,3-3'}=\frac{1}{2}\left(F_{2,3-2',3'}+\frac{f}{b+f}F_{3-3'}-\frac{b}{b+f}F_{2-2'}\right)$	(A51)
$F_{1,2-2',3'}=\frac{1}{2}\left(\frac{d}{e+b}F_{1,2,3-1',2',3'}-\frac{e}{e+b}F_{1-1'}+\frac{b}{e+b}F_{2-2'}-\frac{f}{e+b}F_{3-3'}\right)$	(A52)
$F_{2,3-1',2'}=\frac{1}{2}\left(\frac{d}{b+f}F_{1,2,3-1',2',3'}-\frac{e}{b+f}F_{1-1'}+\frac{b}{b+f}F_{2-2'}-\frac{f}{b+f}F_{3-3'}\right)$	(A53)
$F_{1,2-1',2',3'}=\frac{1}{2}\left(\frac{d}{e+b}F_{1,2,3-1',2',3'}+F_{1,2-1',2'}-\frac{f}{e+b}F_{3-3'}\right)$	(A54)
$F_{2,3-1',2',3'}=\frac{1}{2}\left(\frac{d}{b+f}F_{1,2,3-1',2',3'}+F_{2,3-2',3'}-\frac{e}{b+f}F_{1-1'}\right)$	(A55)
$F_{1,2,3-1'}=\frac{1}{2}\left(F_{1,2,3-1',2',3'}-\frac{b+f}{d}F_{2,3-2',3'}+\frac{e}{d}F_{1-1'}\right)$	(A56)
$F_{1,2,3-2'}=\frac{1}{2}\left(\frac{e+b}{d}F_{1,2-1',2'}+\frac{b+f}{d}F_{2,3-2',3'}-\frac{e}{d}F_{1-1'}-\frac{f}{d}F_{3-3'}\right)$	(A57)
$F_{1,2,3-3'}=\frac{1}{2}\left(F_{1,2,3-1',2',3'}-\frac{e+b}{d}F_{1,2-1',2'}+\frac{f}{d}F_{3-3'}\right)$	(A58)
$F_{1,2,3-1',2'}=\frac{1}{2}\left(F_{1,2,3-1',2',3'}+\frac{e+b}{d}F_{1,2-1',2'}-\frac{f}{d}F_{3-3'}\right)$	(A59)
$F_{1,2,3-2',3'}=\frac{1}{2}\left(F_{1,2,3-1',2',3'}+\frac{b+f}{d}F_{2,3-2',3'}-\frac{e}{d}F_{1-1'}\right)$	(A60)

ПРОИЗВОДНИ ИЗГЛЕДНИ ФАКТОРИ МЕЖДУ УСПОРЕДНИ И НЕУСПОРЕДНИ ПРАВОЪГЪЛНИ ПОВЪРХНОСТИ

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Ключови думи: *радиационен топлообмен, изгледни фактори, отразена радиация*

РЕЗЮМЕ

Определянето на изгледните фактори между две повърхности е важен проблем в радиационния топлообмен за много приложения в строителната физика. Те се използват за определяне на енергията, която се обменя между различните строителни повърхности. Такива примери са излъчващите и поглъщащи топлина стени на сгради, тавани и подове, фотоволтаични панели, хоризонтални и наклонени покриви. Има различни подходи за решаване на този проблем – някои от тях са аналитични, други – числени. Изследваните повърхности могат да бъдат успоредни или неуспоредни, които имат или нямат обща пресечна линия. Някои основни изгледни фактори за различни геометрични конфигурации са включени в онлайн или печатни каталози. Други изгледни фактори на изгледа могат да бъдат определени на база на основните изгледни фактори с помощта на т.нар. View Factor Algebra, която включва някои основни зависимости между изгледните фактори. Целта на тази статия е да изведе и предостави формули за определяне за най-необходимите производни фактори между паралелни и не-паралелни правоъгълни повърхности. Включени са и кодовете на няколко VBA функции за изчисляване на разглежданите базови и производни изгледни фактори.

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