ГОДИШНИК НА УНИВЕРСИТЕТА ПО АРХИТЕКТУРА, СТРОИТЕЛСТВО И ГЕОДЕЗИЯ
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JERK AT THE COMBINED RESPONSE SPECTRUM GRAPH

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Keywords: *jerk, earthquake, response spectrum, combined spectrum, fractional derivate, miamisi*

ABSTRACT

Jerk is the rate of change of acceleration. It is a vector quantity and its scalar magnitude is also the third derivative of position of a body or joint of a structure. Its dimension is [length/time^3]. This paper proposes formulas and presents graphs for jerk response spectrum for a given earthquake ground motion with different damping ratios. A few variants of combined response spectrum graph are shown. All graphs include the jerk response spectrum. The fractional derivative theory was used to improve the combined spectrum representation. A parameter (named miamisi) for assessing the size of the earthquake impact, incorporating the displacement and acceleration, is suggested.

1. Introduction

Jerk is a vector, and therefore it has direction and scalar magnitude, whose SI units are m/s^3 (or m·s^{-3}). Excessive "jerky motion" may result in an uncomfortable stay in buildings, bridges, ride on trains, trams and it should be designed so as to reduce the influence of jerk. The study on jerk and its response spectra will improve the knowledge of the dynamic behaviour of the structures.

Sometimes other names (jolt, surge, terza, time derivative of acceleration – TDoA) are used for this term, but jerk is the most common and therefore preferred for rate of change of acceleration. Jerk is also recognised in the international standards [8].

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The case of free vibrations of SDOF systems including the influence of third derivation of motion is solved in [10] by using third order differential equation. The classical topic in structural dynamics – the response of SDOF systems to harmonic excitation is resolved in [11] with the inclusion of the influence of jerk in the equation of motion. The last work provides an expression of the dynamic magnification factor [2] (or "deformation response factor" [1]) as a function of the damping ratio and of another dimensionless ratio, proportional to the jerk. With these expressions, many dynamics tasks can be solved taking into account jerk, including the expression of logarithmic decrement, used for Free Vibration Decay Method.

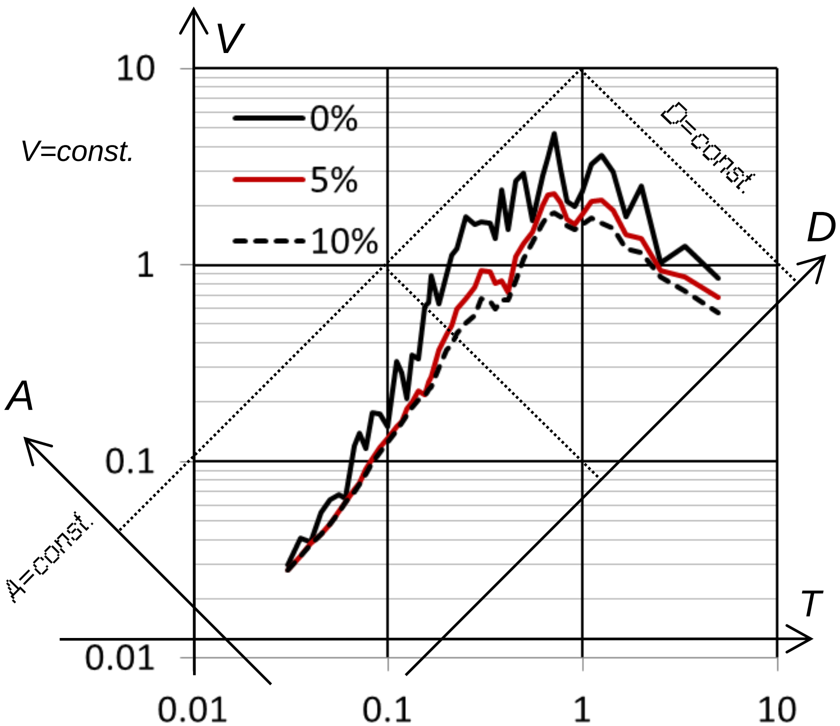

Figure 1. Pseudo-velocity response spectra for Newhall (LA County Fire Station) from 1994 Northridge Earthquake, log scale

Jerk is important when evaluating the destructive effect of motion on a mechanism or the discomfort caused to passengers in a vehicle. The movement of delicate instruments needs to be kept within specified limits of jerk as well as acceleration to avoid damage. When designing a train the engineers will typically be required to keep the jerk less than 2 metres per second cubed for passenger comfort. In the aerospace industry they even have such a thing as a jerkmeter – an instrument for measuring jerk [4]. A vibration serviceability assessment of footbridges is required to evaluate the dynamic response of the footbridges to human-induced excitation [15]. Direct measurement of jerk effects from pedestrians has not yet been performed.

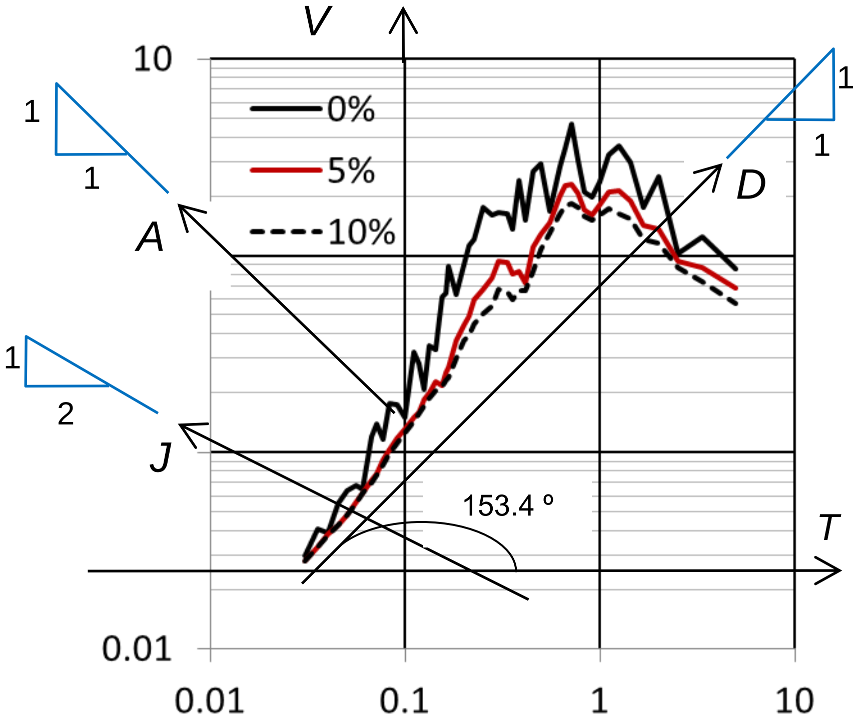

Figure 2. Pseudo-velocity response spectra from 1994 Northridge Earthquake with Jerk axis, log scale

2. Jerk and Jerk Spectrum

The concept of the earthquake response spectrum was introduced by M. A. Biot in 1932. Let's consider deformation response spectrum D, which is the peak value of deformation for a SDOF with a given natural period T and fixed value of the damping ratio ξ:

$$D = \max_t |u(t,T,\xi)| . \tag{1}$$

The quantity V is the peak relative pseudo-velocity. V has a prefix pseudo, because it is not equal to the peak velocity [1]. V is related to D by:

$$V = \frac{2\pi}{T} D . \tag{2}$$

The pseudo-velocity response spectrum is a plot of V as a function of T. The quantity A is peak pseudo-acceleration and it is related to V by:

$$A = \frac{2\pi}{T} V , \tag{3}$$

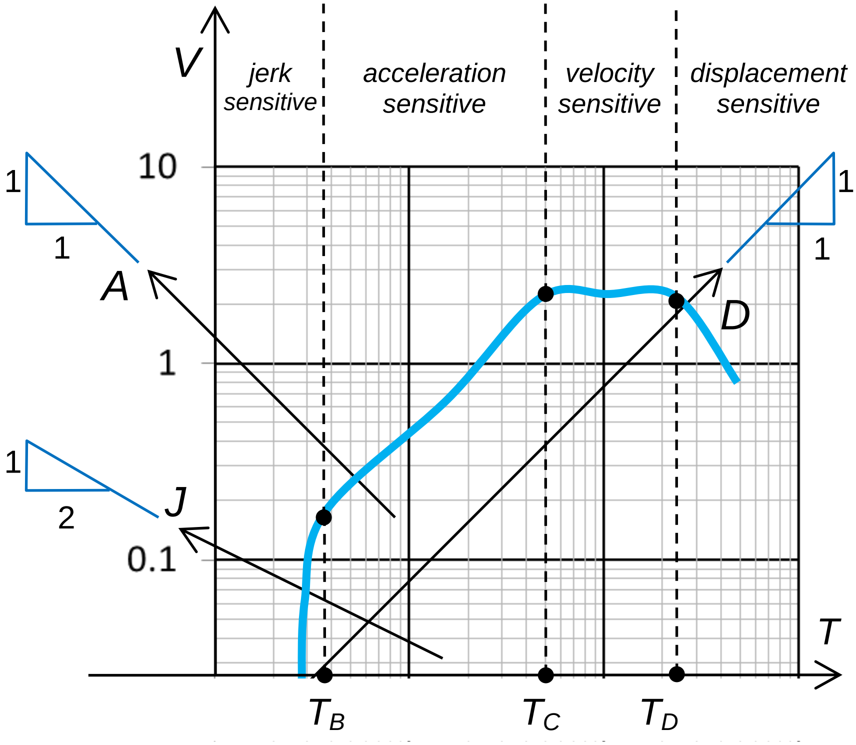

Figure 3. Smooth version of pseudo-velocity response with four spectral regions

The pseudo-acceleration response spectrum is a plot of A as a function of T. The fourth quantity is J – peak pseudo-jerk and it's relation to A is:

$$J = \frac{2\pi}{T} A. \tag{4}$$

The peak pseudo-jerk can be expressed without using the natural period by:

$$J = \frac{VA}{D}. \tag{5}$$

The pseudo-jerk response spectrum is a plot of J as a function of T. J can be calculated also by:

$$J = \left(\frac{2\pi}{T}\right)^2 V = \left(\frac{2\pi}{T}\right)^3 D. \tag{6}$$

For the purpose of drawing the combined response spectrum graph is used:

$$\begin{aligned} \log\ D &= \log V + \log T - \log 2\pi; \\ \log\ A &= \log V - \log T + \log 2\pi; \\ \log\ J &= \log V - 2\log T + 2\log 2\pi. \end{aligned} \tag{7}$$

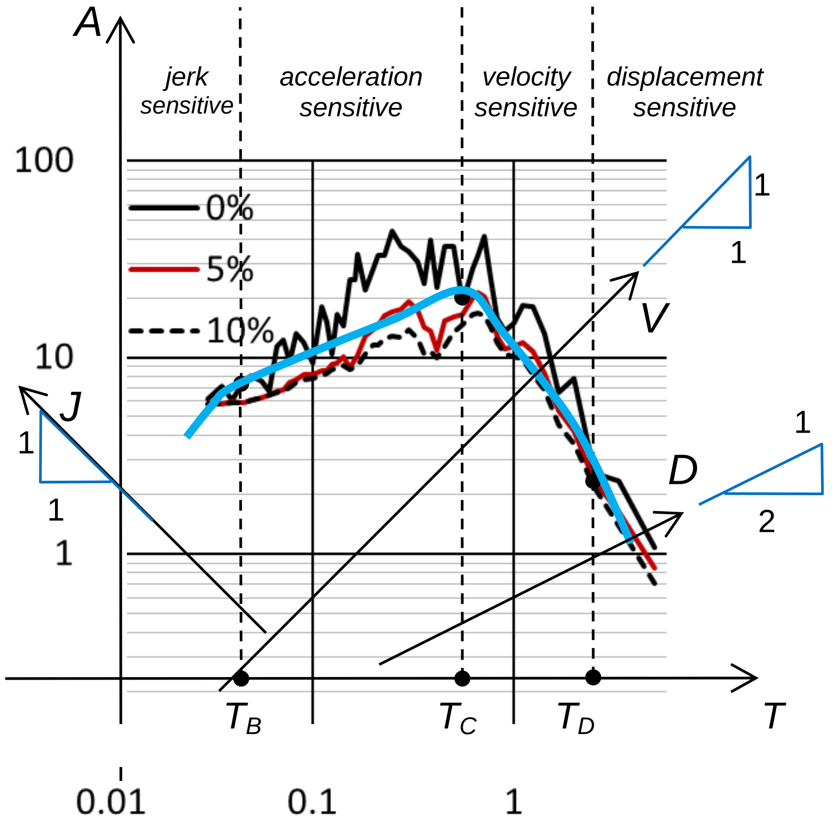

Figure 4. Idealized version of pseudo-acceleration response spectra for 1994 Northridge Earthquake, with four spectral regions

It uses a logarithmic scale for better view and it is obvious that for the lowest values of natural period (less than 0.03 sec.) the jerk is greater than 1000 m/s^3 and tends to infinity for zero periods in the abscissa.

Usually the spectrum is used to find the peak value of equivalent static force developed in the mass of a SDOF:

$$f_{s,\max} = k \times D = m \times A, \tag{8}$$

The four-way plot was developed apparently for the first time by A. S. Veletsos and N. M. Newmark in 1960. That well-known four-way log plot as shown in Fig. 1 allows all three types of spectra to be illustrated on a single graph. On the same graphics can be calculated the quantity J – peak pseudo-jerk. The two scales for D and A are sloping at +45 and +135 degrees respectively. The scale for J is sloping at about +153.435 degrees, see Fig. 2.

Design spectrum is obtained by averaging several spectra from past ground motions and a procedure for scaling. All design spectra have something in common – three regions with constant displacement, velocity and acceleration respectively. Their confines can be seen in Fig. 4. Period T_C is the period separating the acceleration and velocity-sensitive regions of the spectrum. It is clear that periods T_C and T_D vary with damping.

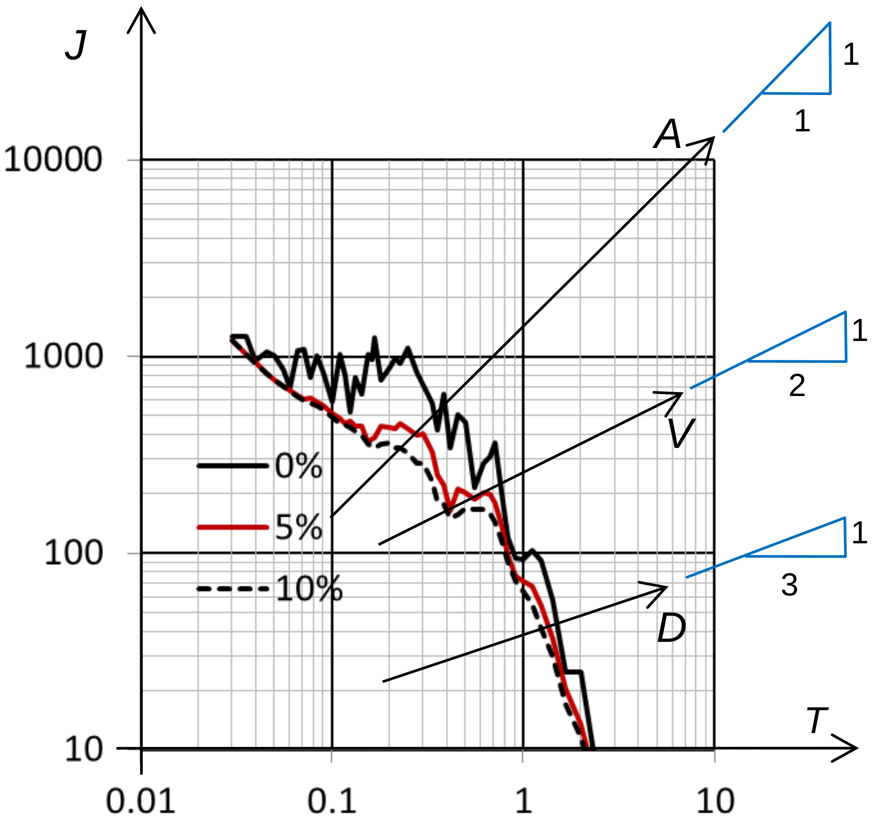

Figure 5. Pseudo-jerk response spectra for 1994 Northridge Earthquake, log scale

By using the function of the pseudo-velocity response spectrum, the other spectra can be obtained by simple algebraic operations. The four quantities of the dynamic motion can be compact presented onto a combined five directions graph. Graphic presentation can take place in different ways.

This idea as shown in Fig. 3 is convenient to use a readymade four-way graph by adding a jerk axis. Now it is obvious that if the natural period is smaller than T_B, it is the jerk sensitive region of the response function.

A combined (five-way) response spectrum graph can be drawn also with T abscissa and A on the ordinate. That variant is shown in Fig. 4 with the slopes of the other response spectrum functions. Usually, the elastic (and design) ground acceleration response spectrum function of T, based on real seismic ground motions in the past, is given in the design codes.

Another option of drawing a combined five-way response spectrum graph is with T on the abscissa and J on the ordinate. That variant is shown in Fig. 5 with the slopes of 1, ½ and ⅓ of the other response spectrum functions. The advantage is that all functions are located in one quadrant. Furthermore, it is clearly visible where the jerk sensitive region is and how big is the difference between the undamped spectrum curve and the other two.

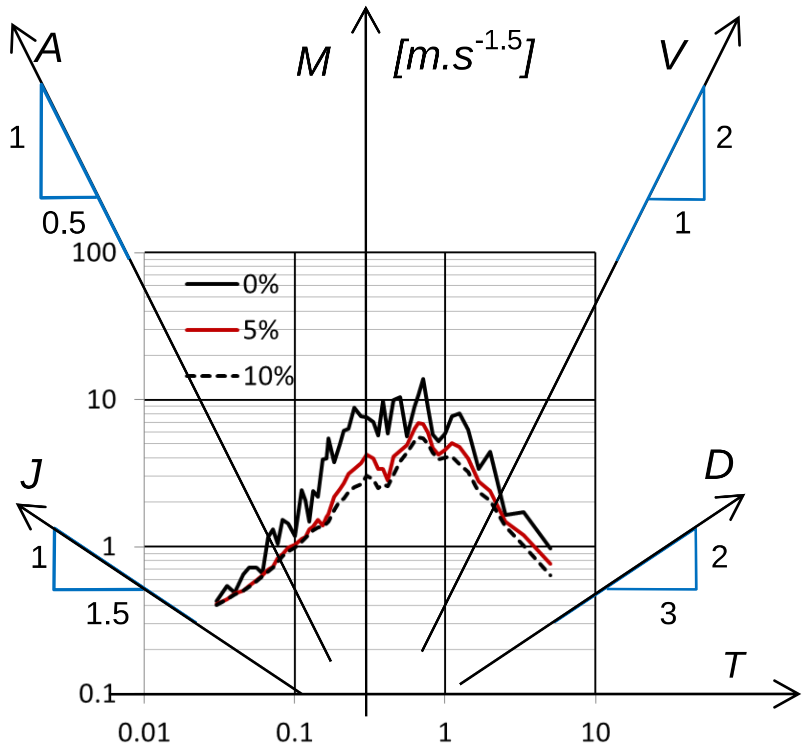

Figure 6. Miamisi response spectra for 1994 Northridge Earthquake, log scale

A beneficial variant (with 6 axis in total) of expressing the response spectrum curve is based on the idea of symmetry. By keeping relationships (7) we can draw spectra as shown in Fig. 6. The axes D and V point to the right. The axes J and A point to the left with the same slopes as the other two. At the vertical axis will be reflected a new quantity noted with M. This drawing format can be named JAMVDT. The proposed name of that new quantity is Miamisi.

3. Miamisi and Miamisi Spectrum

In structural dynamics, we often use Newton's notation for differentiation placing a dot over the function name to represent a time derivative. Velocity is a first derivative of displacement with respect to time. Acceleration is a second derivative of displacement with respect to time. Jerk is a third derivative of displacement with respect to time.

Let's introduce a new quantity: one-and-a-half derivative of displacement with respect to time. This is a fractional derivative used in Fractional Calculus as a derivative of any arbitrary order, real or complex. The birthday of that idea is more than 300 years ago. In a letter dated 30 September 1695, L'Hopital wrote to Leibniz asking him about a particular notation he had used in his publications for the n^{-th} derivative of the linear function [9]. The foundations of the fractional derivative theory were laid by the mathematician Joseph Liouville in a paper from 1832.

That quantity was not used before in structural dynamics and earthquake engineering. For convenience let's assume a name of such quantity: **miamisi**. It has a SI units $\left[m.\sec^{-1.5}\right]$. In Greek "μιάμιση" means "one and a half".

The quantity M is peak relative miamisi. M does not have a prefix pseudo, because it can't be measured with instruments and it is always "pseudo". M is related to D by:

$$M = \left(\frac{2\pi}{T}\right)^{1,5} D \,. \tag{9}$$

The miamisi response spectrum is a plot of M as a function of T. M is related to A by:

$$\begin{gathered} M = A\sqrt{\frac{T}{2\pi}} \\ \log\ M = \log A + \frac{1}{2}\log T - \frac{1}{2}\log 2\pi. \end{gathered} \tag{10}$$

Using a miamisi enables to build symmetrical multi axis response spectrum, as shown in Fig. 6. It brings together four major quantities by:

$$M = \sqrt{V\,A} = \sqrt{D\,J}\ . \tag{11}$$

It is important to know the maximum value of miamisi whose natural period T corresponds to. It is a well-known fact that response spectra functions of many earthquakes are the main data base to create design spectra in design codes. In the contemporary design codes the maximum value of (elastic) miamisi is for $T = T_C$, because at this point both velocity and acceleration have their maximal plateau value. The point C is very important in the earthquake design codes, because it is the border between short (and medium) period and long period structures. In engineering practice the most important parameters of a given DOF in structural dynamics are the acceleration and displacement. The miamisi includes both of them with prevail in a ratio of three to one in favor of the acceleration, visible in the form:

$$\begin{gathered} M = A^{\frac{3}{4}}\ D^{\frac{1}{4}} \\ M = \sqrt{A\sqrt{A\,D}}. \end{gathered} \tag{11}$$

With the last formula the spectral miamisi can be easily calculated.

4. Conclusion

The use of a fractional derivative is a new approach in structural dynamics. In general, the fractional calculus will give the opportunity to study better the processes associated with dynamic loading.

The results show that the jerk spectrum has similar rules as acceleration spectrum in general, and the amplitude is relative to the predominant period, especially for structures with short or medium period [6].

A new dimension, called miamisi, has been developed. A different graphic expression of the spectral miamisi is fully described in this paper. The application of jerk and miamisi spectra in conjunction with the normal building design is also needed.

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ВЕЛИЧИНАТА ДЖЪРК ВЪРХУ КОМБИНИРАНАТА ГРАФИКА НА СПЕКТЪР НА РЕАГИРАНЕ

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Ключови думи: *джърк, земетръс, спектър на реагиране, псевдо-джърк, дробна производна, миамиси, строителна динамика*

РЕЗЮМЕ

Джърк (jerk) се нарича степента на изменение на ускорението. Това е векторна физична величина и нейната скаларна големина е също така третата производна по време на положението на тяло или възел от конструкция. Единицата за джърк е [дължина/време^3] и може да се представи като втора производна на скоростта по време. В публикацията са предложени формули и са представени графики за спектър на реагиране за джърковете за конкретно земетресение при различни коефициенти на относително затихване. Показани са няколко варианта на комбиниран (многоосов) спектър на реагиране. Всички варианти са в логаритмичен мащаб и съдържат оста на джърка. Дробна производна (fractional derivative) е използвана за подобряване на представянето на спектралните графики. предложен е нов параметър, наречен миамиси. Той представлява дробна производна на преместването и може да се използва за оценяване на големината на земетръсното въздействие. Спектралното миамиси може да се изрази чрез спектралните преместване и ускорение.

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