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ACCUMULATION OF THE DISCREPANCIES [D] ALONG THE BALKAN MOUNTAINS CROSSING LINES 26, 31, 35, 37 IN THE THIRD LEVELLING OF BULGARIA

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ABSTRACT

In order to investigate the laws of accumulation of the differences between two independent measurements in precise levelling, four lines which cross the Balkan Mountains have been analyzed. These lines are part of the Third Levelling of Bulgaria.

By using simple graphical visualization techniques, an attempt is made to illustrate the relationship between the square root of the levelling length and the accumulated discrepancies along the lines.

Analyzing the above-mentioned lines, an almost functional relationship between the heaping up of the double differences and the absolute values of the measured elevations is established.

1. Introduction

The First Order Levelling is commonly applied to engineering and scientific activities where the highest possible accuracy of the vertical movement assessments is necessary, e.g. determining the shape and size of the Earth and its gravity field [1, 2], recent uplift of the Earth's crust [3, 4], applied engineering tasks [5 – 7], etc.

In order to be able to respond to the increasing accuracy requirements, the precise levelling should revise some of the basic assumptions about the laws of the accumulation of the

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differences between both forward and backward measurements of the elevations between nodal benchmarks, especially when lines pass through hilly and mountainous territories [8 – 15].

Without taking into consideration the facts about the relationships between the absolute values of the discrepancies and the sum of the absolute elevations along the levelling routes which were obtained in the Second and Third Levelling of Bulgaria [14, 15, 16] defines the allowable differences between the two measurements of the elevations in the precise levelling lines by equation (1).

$$|D| \leq 2,25mm\sqrt{L}, \quad (1)$$

where L is the length of the line in km.

The above criterion has two disadvantages, which are explained below.

1.1. Irrelevance of the coefficient

Analyzing [17] in part of the Third Levelling of Bulgaria, one can find that 18 out of 55 lines or 33% of all lines were not able to satisfy this requirement. The coefficient of 2,25 is unsuitable as a control criterion especially in the mountain crossing lines. According to Figure 1 and Figure 7, this coefficient should be almost twice greater.

1.2. The stereotype of the superior relationship between $|D|$ and the square root of the length of the line

Since the dawn of the precise levelling, it has been believed that the best factor which describes the accumulation of the errors in the precise levelling is the square root of the levelling distance. This assumption might be true for countries with a flat relief but is implausible and nonsensical for states with hilly and mountainous territories.

Starting with equation (2), which gives a common station elevation between two points i and $i+1$, the final elevation between the nodal benchmarks A and B can be given by (3).

$$h_{i,i+1}^{st.} = b_{i+1} - a_i. \quad (2)$$

$$H_{A,B} = \sum_{i=2}^n b_i - \sum_{i=1}^{n-1} a_i. \quad (3)$$

Based on the above logic, the final elevation between the nodal benchmarks B and A can be written by (4).

$$H_{A,B} = \sum_{i=2}^{\kappa} b_i - \sum_{i=1}^{\kappa-1} a_i. \quad (4)$$

Thus, the difference between the two measurements D according to the basic math rules, has to be (5), respectively (6).

$$D_{A,B} = H_{A,B} + H_{B,A}. \quad (5)$$

$$D_{A,B} = \sum_{i=2}^n b_i - \sum_{i=1}^{n-1} a_i + \sum_{i=2}^{\kappa} b_i - \sum_{i=1}^{\kappa-1} a_i. \quad (6)$$

The question is: Where in equation (6) does the square root of the levelling distance take part?

One might say that the above logic is a narrow point of view, but it is based on a simple math. Let them not face such a situation where, looking at the Earth's shadow on the Moon, will claim that the Earth's surface is flat. The understanding of the accumulation of the discrepancies in precise levelling pass through solid and correct basic logic not through assumptions and popular beliefs.

Analyzing (6) it is crystal clear that the difference $D_{A,B}$ is a function of the sum of the station elevations along the line and their compensations. Moreover, the difference $D_{A,B}$ can be associated as a misclosure of the loop which starts in point A, passes through point B and returns in point A. Sometimes the value of $D_{A,B}$ increases along the line but sometimes decreases due to some compensation of the effect of some error factors. As a result, some lines and loops have discrepancies or misclosures close to zero despite their long length, e.g. the ring loop in the Third Levelling of Bulgaria, whose length is 2550,18 km, has only 2,85 mm misclosure.

On the opposite side is line 26 whose discrepancy is almost 40 mm per 50 km length.

Looking at equation (6) it seems more reasonable and scientific that the difference $D_{A,B}$ should be defined as a function of the absolute values of the measured elevations on the station. Therefore, (7) or better (8) should be preferred to (1), especially for mountain crossing lines.

$$|D| = b \sum_{i=1}^m |h_m^{section}|, \quad (7)$$

$$|D| = a + b \sum_{i=1}^m |h_m^{section}|. \quad (8)$$

The relevance of the above thoughts is proved by examples based on [17].

2. Discrepancies in line 26

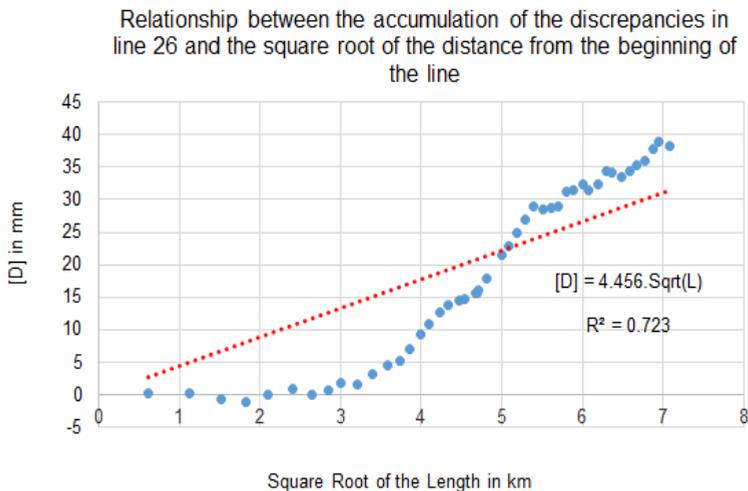


Figure 1. Relationship between [D] and the Square Root of the Length in line 26

Relationship between the accumulation of the discrepancies in line 26 and the sum of the absolute elevations from the beginning of the line

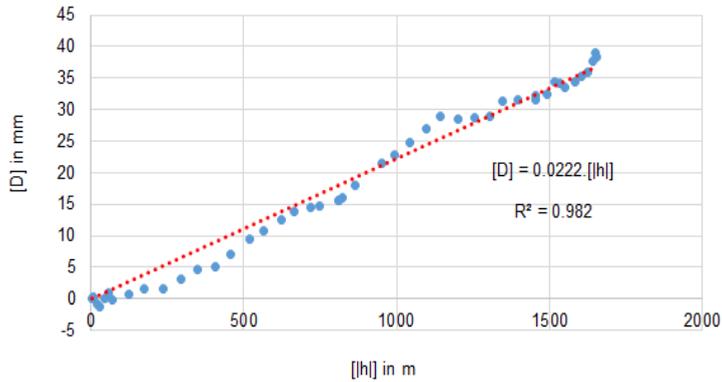


Figure 2. Relationship between [D] and [h] in line 26

Looking at Figure 1 and Figure 2 one can see that:

- A. The accumulation of the double run differences referring to the square root of the length is a curve rather a linear function (Fig. 1).
- B. The coefficient 4,456 in Figure 1 does not correspond with the coefficient 2,25 in (1).
- C. The coefficient of determination R^2 in the classic approach is 26% less than the coefficient of determination pictured in Figure 2.
- D. Almost a functional relationship has been found between the accumulation of the discrepancies [D] and $\sum|h|$.

This line is widely analyzed in [13], where the accumulation of [D] is explained by non-vertical positions of the rods during measurements.

3. Discrepancies in line 31

Relationship between the accumulation of the discrepancies in line 31 and the square root of the distance from the beginning of the line

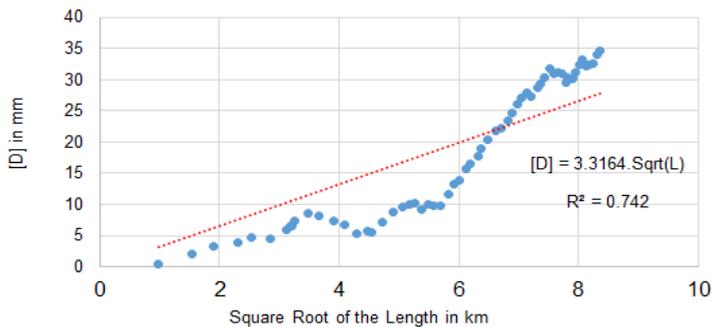


Figure 3. Relationship between [D] and the Square Root of the Length in line 31

Relationship between the accumulation of the discrepancies in line 31 and the sum of the absolute elevations from the beginning of the line

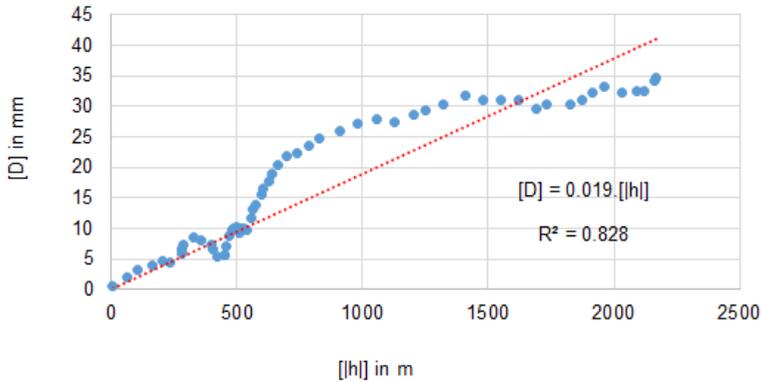


Figure 4. Relationship between [D] and [[h]] in line 31

Looking at Figure 3 and Figure 4 it is obvious that:

- A. The accumulation of the double run differences referring to the square root of the length is a curve rather a linear function (Fig. 3).
- B. The coefficient 3,3164 in Figure 3 does not correspond with the coefficient 2,25 in (1).
- C. The coefficient of determination R^2 in the classic approach is 9% less than the coefficient of determination depicted in Figure 4.

4. Discrepancies in line 35

Relationship between the accumulation of the discrepancies in line 35 and the square root of the distance from the beginning of the line

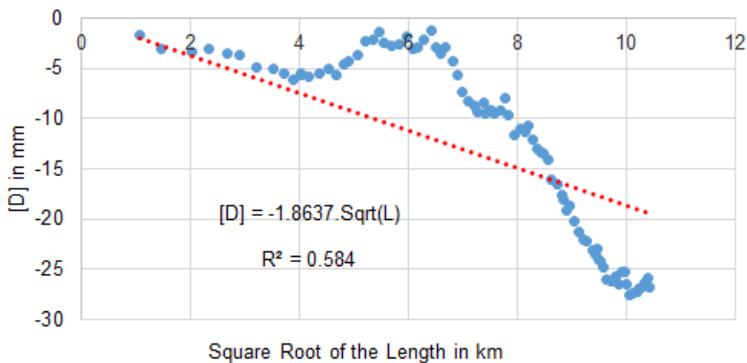


Figure 5. Relationship between [D] and the Square Root of the Length in line 35

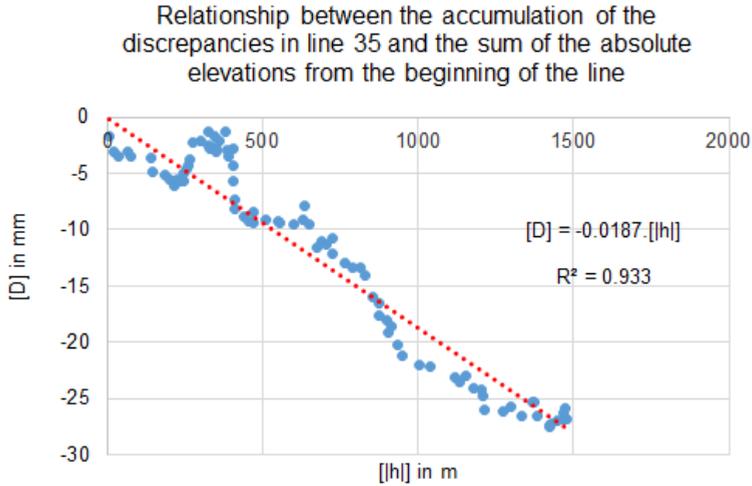


Figure 6. Relationship between [D] and [h] in line 35

Looking at Figure 5 and Figure 6 one can see that:

- A. The accumulation of the double run differences referring to the square root of the length is a curve rather than a linear function (Fig. 5).
- B. The coefficient of determination R^2 in the classic approach is 35% less than the coefficient of determination pictured in Figure 6.
- C. Almost a functional relationship has been found between the accumulation of the discrepancies [D] and $\sum|h|$.

5. Discrepancies in line 37

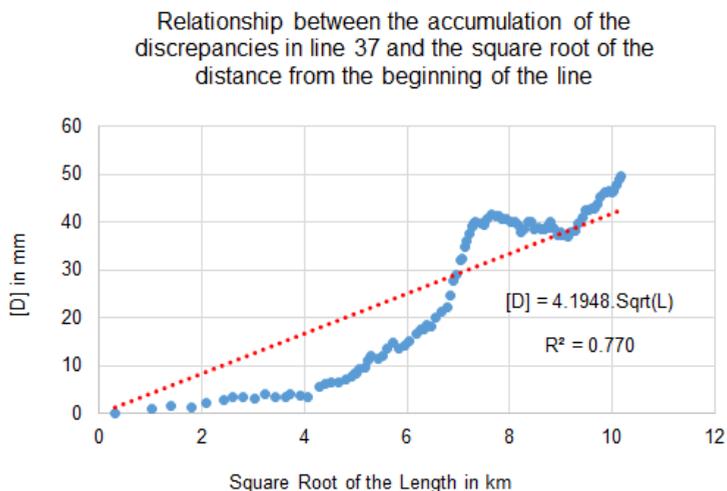


Figure 7. Relationship between [D] and the Square Root of the Length in line 37

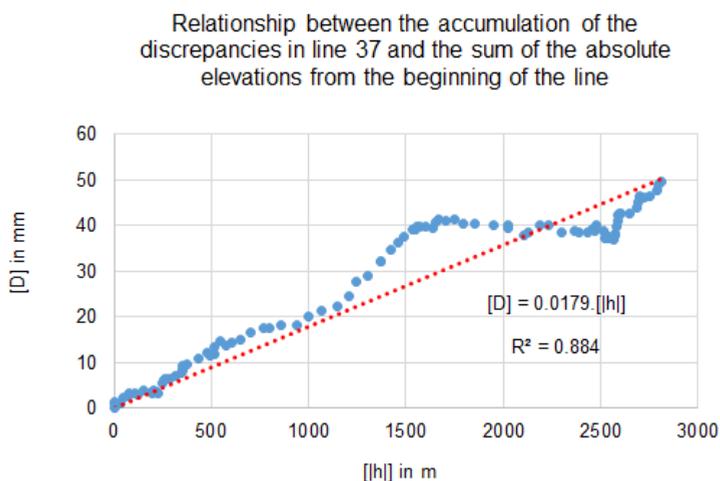


Figure 8. Relationship between [D] and [h] in line 37

Looking at Figure 7 and Figure 8, it is obvious that:

- A. The accumulation of the double run differences referring to the square root of the length is a curve rather than a linear function (Fig. 7).
- B. The coefficient 4,1948 in Figure 7 does not correspond with the coefficient 2,25 in (1).
- C. The coefficient of determination R^2 in the classic approach is 11% less than the coefficient of determination pictured in Figure 8.

6. Conclusion

Look deep into nature, and then you will understand everything better [18]. This thought of Einstein's is fully relevant concerning the precise levelling in Bulgaria. Looking at the coefficients of the relationships between [D] and [h] in Figures 2, 4, 6 and 8, one can see that the absolute values of these coefficients are in the range from 0,0187 to 0,0222. This fact shows that there is a 0,02 mm accumulation of discrepancies per measured meter elevation considering the Balkan Mountains crossing lines 26, 31, 35 and 37. Attention should be paid to the fact how close to each other these coefficients are, which is opposite to the situation concerning the coefficients pictured in Figures 1, 3, 5 and 7.

It is a well-known fact that in the routes which contain higher inclinations the influence of the levelling refraction [12], non-vertical position of the levelling rods during the measurements [13], the errors in the rod meters, etc. deteriorate the levelling accuracy. In order to understand deep inside these errors, further investigations on training polygons are necessary [19]. Involving the new digital levels in the precise levelling process [16, 20] demands the construction of one or more modern comparators for automated calibration of both classic and barcode levelling rods [21, 22].

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НАТРУПВАНЕ НА РАЗЛИКИТЕ ОТ ДВОЙНИТЕ ИЗМЕРВАНИЯ НА ПРЕСИЧАЩИТЕ СТАРА ПЛАНИНА ЛИНИИ 26, 31, 35 И 37 ОТ ТРЕТОТО ИЗМЕРВАНЕ НА ПЪРВОКЛАСНАТА НИВЕЛАЧНА МРЕЖА НА Р БЪЛГАРИЯ

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Ключови думи: прецизна нивелация, грешки, разлики, тежести при изравнение

РЕЗЮМЕ

С цел изучаване на законите на натрупване на разлики между двойните измервания в прецизната нивелация четири линии, пресичащи Стара Планина, са анализирани. Тези линии са част от третата първокласна нивелация на Р България.

Използвани са опростени графични техники за визуализация и е направен опит за изобразяване на взаимовръзката между квадратния корен на дължината на пронивелираното разстояние и натрупаните двойни разлики между двете нивелирания.

Чрез анализ на въпросните линии бе установена една почти функционална връзка между натрупването на двойните разлики и натрупването на абсолютните стойности на измерените превишения.

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