ГОДИШНИК НА УНИВЕРСИТЕТА ПО АРХИТЕКТУРА, СТРОИТЕЛСТВО И ГЕОДЕЗИЯ
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AN ATTEMPT AT EXPLAINING THE ABSOLUTE DISCREPANCIES |*D*| ACCUMULATED BETWEEN THE FUNDAMENTAL BENCH MARKS OF THE FIRST AND THE SECOND ORDER IN THE THIRD LEVELLING OF BULGARIA BY SOME REGRESSION MODELS

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Keywords: *precise levelling, errors, discrepancies, differences*

ABSTRACT

In order to investigate the influence of different factors on the accumulation of the discrepancies in the precise levelling, three regression models are compared.

Using their residual plots, a heteroscedasticity is detected in that model which is based on the levelling distance.

More homoscedastic plots have been yielded in the model based on the sum of the absolute values of the elevations in the sections.

According to the above results and the minimal square error of the last model, a conclusion of superiority of the measured elevations for forming of the discrepancies over the levelling distance is made.

1. Introduction

The First Order Levelling is the most accurate method for vertical measurements and is commonly used in scientific tasks [1 – 4] and applied geodesy [5 – 7] when the highest possible accuracy is necessary.

In order to respond to the increasing requirements which the precise levelling faces with, the study of the factors that affected its accuracy continues. More often various methods of the statistics are applied [8 – 10].

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One of the preferred techniques is the ordinary least-squares regression [11 – 14]. Using this method, the classic relationships between the discrepancies |*d*| and the square roots of the levelling distances in the sections have been investigated [8]. A step out of beaten track was done by [9], where some models of the discrepancies |*D*| as a function of the length of the levelling lines has been studied.

A suggestion to look in a different direction was made by [15]. A systematic effect of the inclination of the levelling routes were detected [16 – 19]. As a result, some different regression models have been analyzed [10] and weights based on the sum of the absolute section elevations in the levelling lines have been recommended [10, 20]. Also, new allowable differences |*D*| between both the forward and backward measurements in lines have been developed [21].

The current paper is an extension of [10, 21, 22] but this time the focus is on the discrepancies |*D*| between both measurements among fundamental bench marks I and II order.

There are 225 sublines, which are defined by the above-mentioned bench marks. The minimum length is 4,312 km and the maximum one is 77,943 km. The average length is 25,028 km [23].

The reason why the sublines are object of the current investigation is because of the fact that they are the link between the levelling sections and lines. The sections are too short for some systematic accumulation of errors, but the lines are long and along them there are some error compensations.

2. The Model $|D| = aL^{0,5}$

This is the most famous levelling model in the world. Another question is whether this model is the most appropriate for the territory of Bulgaria?

Using least-squares regression and the measured data of the Third Levelling of Bulgaria [11] the coefficient a = 1,237 is yielded. The standard error of this model is determined to be equal to 5,032 mm. Residuals and Absolute Residual Plots are created and pictured in Fig. 1 and Fig. 2, respectively.

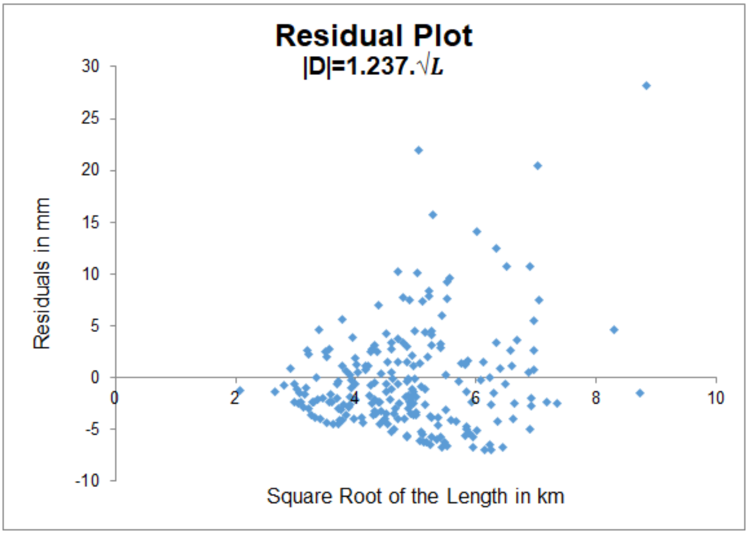

Figure 1. Residuals of the model $|D| = 1,237L^{0,5}$

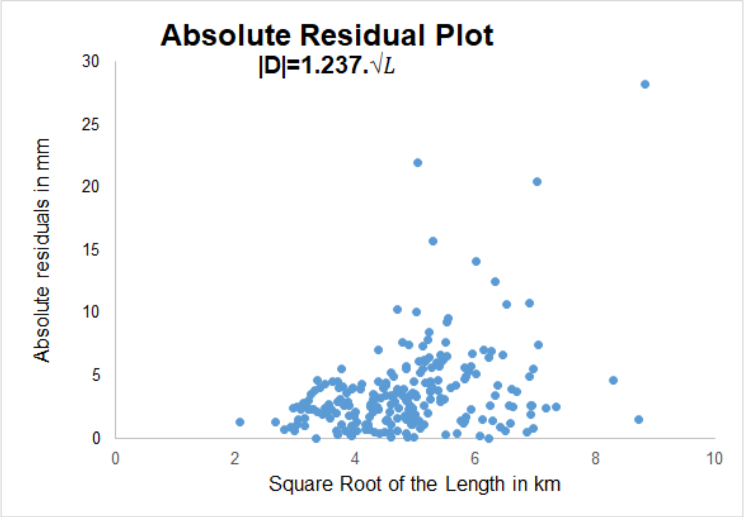

Figure 2. Absolute residuals of the model |*D*| = 1,237*L*^{0,5}

Looking at Fig. 1 and Fig. 2 one can see well-expressed heteroscedasticity or unequal dispersion of the residuals. This fact clearly shows that the square root of the levelling distance is not the variable in the regression model which explains $|D|$ in the best way because of the failing of the basic linear regression assumptions [11 – 14, 24]. Moreover, the presence of unequal dispersion of the residuals is a serious obstacle for building reasonable confidence bands, further on for allowable limits of the discrepancies $|D|$.

There are some recommendations for solving this issue. One decision is the usage of weighted least-squares regression [11, 24]. This approach will be applied in a further publication. This paper will discuss two other techniques to cope with the detected heteroscedasticity in the model $|D| = aL^{0,5}$. These fixes are the first two approaches listed on [12]. Let us start with the first one.

3. The Model $|D| = aL^b$

In fact, the model $|D| = aL^b$ is a nonlinear model but it can easily turn into linear form by taking a natural logarithm of its both sides (1) – (2).

$$\ln|D| = \ln\left(aL^b\right), \tag{1}$$

$$\ln|D| = \ln a + b\ln L, \tag{2}$$

Now, by applying an ordinary least-squares regression, it is easy to yield the coefficients a and b in (2). According to the used data [11], a = 0,39311 and b = 1,05757. This fact reclaims the conclusions which were made in [9] where the First and the Second Levelling of Bulgaria were analyzed and as a result, the model $|D| = 0{,}33L^{0,92}$ was obtained.

The standard error of the model $|D| = 0{,}39311L^{1,05757}$, which is produced with the data of sublines in the Third Levelling of Bulgaria, is estimated of 8,953 mm. The last value is greater than the standard error of the model from section 2 – $|D| = 1{,}237L^{0,5}$. What is more, there is not any proceeding with the heteroscedasticity, as can be seen in Fig. 3 and Fig. 4.

Consequently, a different variable in the regression model should be used.

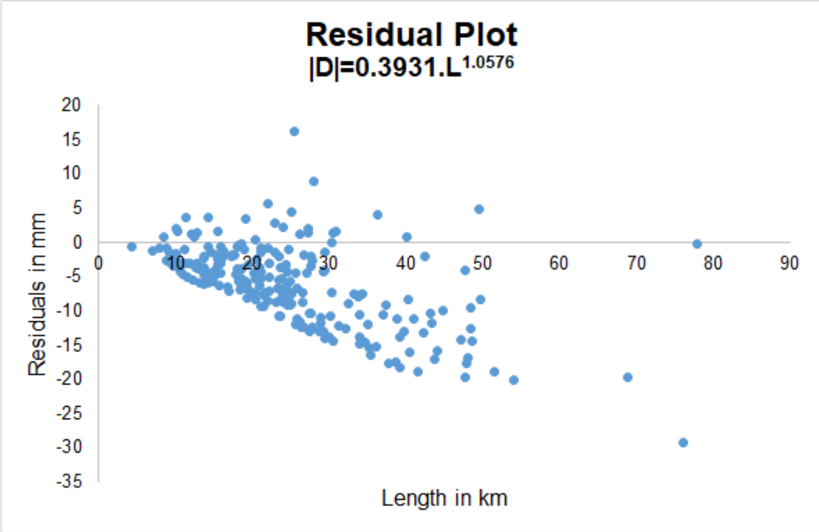

Figure 3. Residuals of the model $|D| = 0{,}3931L^{1{,}0576}$

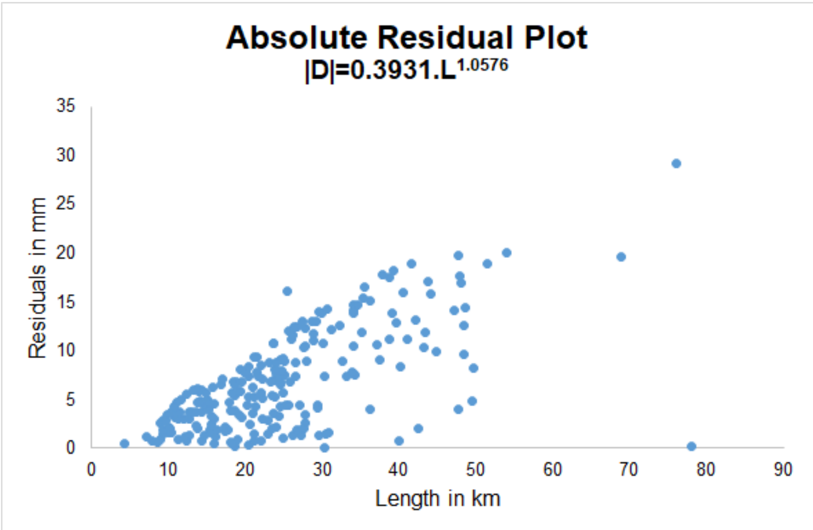

Figure 4. Absolute residuals of the model $|D| = 0{,}3931L^{1{,}0576}$

4. The Model $|D| = a + b[|h|]$

The model $|D| = a + b[|h|]$ derives from (3).

$$D_{A,B} = \sum_{i=2}^{n} b_i - \sum_{i=1}^{n-1} a_i + \sum_{i=2}^{k} b_i - \sum_{i=1}^{k-1} a_i \ . \qquad (3)$$

The denotes b, a, n and k in equation (3) are the backward rod reading, the forward rod reading, the number of the backward and the forward levelling stations along the line, respectively.

Utilizing the Third Levelling of Bulgaria data, the regression model $|D| = 2{,}653 + 0{,}0079[|h|]$ has been produced. The standard error of this model is equal to 4,814 mm, which is the least standard error of the models analyzed here. The more important thing is a step ahead in solving the inequality of the residual dispersion – Fig. 5 and Fig. 6.

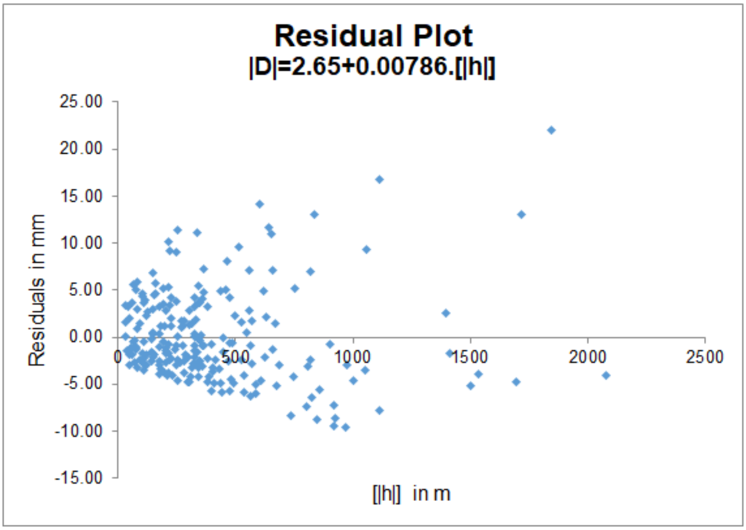

Figure 5. Residual of the model $|D| = 2{,}65 + 0{,}00786[|h|]$

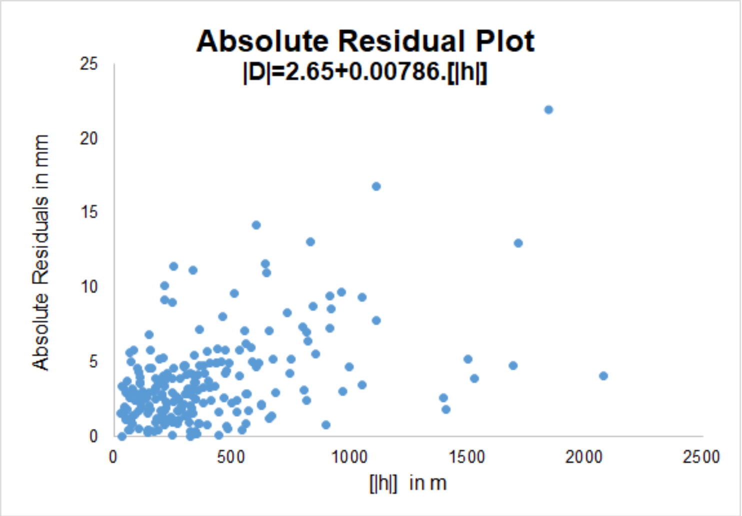

Figure 6. Absolute residual of the model $|D| = 2{,}65 + 0{,}00786[|h|]$

Actually, if one uses a self-weighted regression [25], a better regression model with standard error equal to 3,858 mm and expression $|D| = 2{,}006 + 0{,}0085[|h|]$ will be yielded.

Looking at Fig. 5 and Fig. 6 one can see more homoscedastic spread of the residuals than in Fig. 1 – Fig. 4. Also, it can be seen that the value of the greatest residual is 22 mm against 28 mm and 29 mm of the first and the second analyzed models, respectively.

All these facts explicitly illustrate that the discrepancies $|D|$ in the sublines in the Third Levelling of Bulgaria are explained in the best way as a function of the sum of the absolute values of the measured section elevations. Based on this fact and on the results obtained in [10] concerning the lines in the Second and the Third Levelling in Bulgaria, a revision of [26] in part of the allowable differences in lines, the preliminary assessment of the accuracy and the choice of the weights applied in the adjustment of the precise levelling is needed. Also, allowable differences of the discrepancies of the sublines should be added. A reasonable one can be based on the discussed model in this section.

5. Conclusion

It is high time to face with the fact that the popular belief that the square root of the levelling distance is the variable which describes the discrepancies $|D|$ in the best way is not appropriate concerning the relief of Bulgaria.

If we want to develop the precise levelling, then more time and efforts must be invested in learning the errors referred to the factor of elevation, e.g. vertical movements of the tripod and rods during the measurements, change in the rod meter, calibration of the rods, systematic effect of the levelling refraction, etc.

The suggestions given in [27 – 28] should also be taken into consideration.

REFERENCES

1. *Tsanovski, Y.* GNSS izmervania za nuzhdite na precizni visochinni opredelenia v R Balgaria, Godishnik na Universiteta po arhitektura, stroitelstvo i geodezia – Sofia, 2021, tom 54, br. 2, 211-217, ISSN 1310-814X.

2. *Lambeva, T., Peneva, E., Gospodinov, S.* Normal height and geopotential number differences determination for the territory of Bulgaria with use of data from global gravity field models. 19th International Multidisciplinary Scientific GeoConference SGEM 2019. Conference proceeding, 2019, 309-318.

3. *Gospodinov, S., Stereva, K.* Determining of areas on the territory of R Bulgaria with a low intensity of the recent vertical movements of the Earth's crust. 20th International Scientific Multidisciplinary Conference on Earth and Planetary Sciences SGEM2020, Albena, Bulgaria.

4. *Belyashky, T.* Izsledvane na savremennite vertikalni dvijenia na zemnata kora po nivelachnata linia Vidin – Mihajlovgrad. Geodezia, kartografia i zemeustrojstvo, 1987, 2: 7-9.

5. *Peneva, E., Gospodinov, S., Lambeva, T., Dimeski, S.* Preliminary results of connection between two State leveling networks via cross-border levelling measurements. 19th International Multidisciplinary Scientific GeoConference SGEM 2019. Conference proceeding, 2019, 257-266.

6. *Angelov, A.* Geodezicheski metodi za izsledvane na deformacionni procesi pri visoki sgradi i inzhenerni saorazhenia. UACEG, 2017, https://uasg.bg/filebank/att_13033.pdf, poseten na 16.12.2022.

7. *Angelov, A., Penev, P.* Preliminary assessment of the accuracy of the tunnel engineering-geodetic network. // Godishnik na UASG, 2017, 50 (2): 251-259.

8. *Avdzhiev, M.* Statisticheski analiz na greshkite v preciznata nivelatsia. Disertatsionen trud, 1979.

9. *Ivanov, E.* Matematiko-statisticheski analiz na parvoklasni nivelachni izmervania na teritoriyata na Balgaria, Balgarska Akademia na Naukite, Tsentralna laboratoria po visha geodezia, 1975, 1:22-39.

10. *Cvetkov, V.* Vazmozhnosti za povishavane na tochnostta na visokotochnata geometichna nivelacia v Bulgaria. Disertatsionen trud, UACG, Geodezicheski fakultet, katedra „Vissha geodezia“, 2005.

11. *Draper, N., Smith, H.* Applied Regression Analysis, Second Edition, John Wiley & Sons, 1981.

12. *Mooi, E., Sarstedt, M.* A Concise Guide to Market Research. The Process, Data, and Methods Using IBM SPSS Statistics, Springer – Verlag Berlin Heidelberg, 2011, ISBN 978-3-642-12541-6.

13. *Rawlings, J., Pantula, S., Dickey, D.* Applied Regression Analysis: A Research Tool, Second Edition, Springer – Verlag New York, 1998, ISBN 0-387-98454-2.

14. *Ramachandran, K., Tsokos, Chr*. Mathematical Statistics with Applications, Elsevier, 2009, ISBN 978-0-12-374848-5.

15. *Boyadjiev, B.* Sastoyanie i razvitie na darzhavnata nivelachna mrezha na NR Balgaria, Geodezia, kartografia I zemeustrojstvo, 1985, 2: 12-15.

16. *Gekov, D., Gospodinov, S.* Otnosno obrabotkata na rezultatite ot visokotochnata geometricna nivelacia za celite na geodinamicnite izsledvania, Geodezia, kartografia, kadastar, 1988, 6.

17. *Gekov, D., Gospodinov, S.* Vlianie na naklona na terena varhu poluchenite skorosti na vertikalnite dvizhenia na zemnata kora u nas. Geodezia, kartografia, kadastar, 1989, 2.

18. *Gekov, D., Gospodinov, S.* Sistematichno vlianie na naklona na terena varhu skorostite na tochki ot zemnata kora ot gledna tochka na tehnologiata na izmervane. Geodezia, kartografia, kadastar, 1989, 3-4.

19. *Gekov, D., Gospodinov, S., Zdravchev, I.* Za otchitane na refrakciata pri visokotochnata geometrichna nivelacia. Geodezia, kartografia, kadastar, 1989, 1.

20. *Gekov, D., Tzvetkov, V.* The Problem About Choice of Weights in the Adjustment of Precise Leveling Networks. International symposium on “Modern technologies, education and professional practice in geodesy and related fields”, Sofia, 03-04 November 2005, p. 290-296.

21. *Gekov, D., Tzvetkov, V.* Limits for Differences in Values of Forward and Backward Heights Between Nodal Bench Marks in Levelling Lines. International symposium on “Modern technologies, education and professional practice in geodesy and related fields”, Sofia, 03-04 November 2005, p. 297-304.

22. *Kunchev, I.* On standards and specifications for vertical control surveys. // Godishnik na UASG, 2021, 54 (2): 253-262.

23. Geokartfond – nivelachen arhiv.

24. https://en.wikipedia.org/wiki/Heteroscedasticity, poseten na 28.12.2021.

25. https://center-based-statistics.com/html/linearReg.html, poseten na 02.02.2022.

26. Instruktsia № RD-02-20-1 ot 15 yanuari 2021 g. za sazdavane i poddarzhane na darzhavnata nivelachna mrezha. V sila ot 05.02.2021 g., Izdadena ot ministara na regionalnoto razvitie i blagoustroystvoto, Obn. DV, br. 10 ot 5 Fevruari 2021 g.

27. *Gospodinov, S., Belyashki, T., Peneva, E.* Analiz na darzhavnata nivelachna mrezha na Republika Balgaria. Doklad na rabotna grupa po zadaca 3. Syzdavane na programa i usavarshenstvane na darzhavnite geodezicheski mrezhi (3.2. Darzhavna nivelachna mrezha) kym Saveta po geodezia, kartografia i kadastar, 2004, https://www.cadastre.bg/sites/default/files/documents/SavetPoGeodeziaKK/analiz_nivel_mrega_bg_agkk.pdf, poseten na 29.01.2022.

28. *Gospodinov, S., Peneva, E., Penev, P., Lambeva, T., Tsanovski, Y., Dzhorova, S., Marinov, G., Radev, I.* Savremenni aspekti na geometricnata nivelacia. // Godishnik na Universiteta po arhitektura, stroitelstvo i geodezia – Sofia, 2016, Vol. 49, Issue 4, 9-23, ISSN 1310-814X.

ОПИТ ЗА ОБЯСНЯВАНЕ НА НАТРУПВАНЕТО НА АБСОЛЮТНИТЕ РАЗЛИКИ |*D*| МЕЖДУ ВЕКОВНИТЕ НИВЕЛАЧНИ РЕПЕРИ I И II СТЕПЕН В ТРЕТАТА ПЪРВОКЛАСНА НИВЕЛАЦИЯ НА БЪЛГАРИЯ ЧРЕЗ НЯКОИ РЕГРЕСИОННИ МОДЕЛИ

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Ключови думи: *прецизна нивелация, грешки, разлики*

РЕЗЮМЕ

С цел изследване на влиянието на различни фактори на грешки върху натрупването на разлики от двойните измервания в първокласната нивелация са съставени и сравнени три регресионни модела.

Установено е ясно изразено непостоянство на дисперсията на остатъците на моделите, които са функция на пронивелираното разстояние.

Моделът, базиран на сумата на абсолютните стойности на измерените превишения в нивелачните секции, има най-малка средна квадратна грешка и най-постоянна дисперсия на остатъците.

Въз основа на така получените резултати е направен изводът, че натрупването на разликите от двойните измервания в третата първокласна нивелация на България се свързва по-скоро с натрупване на измерените превишения, отколкото с увеличаване на дължината на нивелачната линия.

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