



Получена: 11.02.2022 г.

Приета: 14.04.2022 г.

ALLOWABLE DISCREPANCIES IN THE PRECISE LEVELLING SECTIONS BASED ON REAL DATA

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Keywords: precise levelling, errors, discrepancies, differences

ABSTRACT

In order to define allowable vertical control limits of the discrepancies in the precise levelling sections, the yielded ones in the Third Levelling of Bulgaria are analyzed.

Using some nonparametric tests, it has been proven that the above-mentioned discrepancies do not follow a normal distribution.

Based on the real distribution of the analyzed discrepancies, an allowable limit of the differences between both measurements of the section elevations in the precise levelling of Bulgaria is proposed.

1. Introduction

An important issue, which any recommendation of precise levelling face with, is defining the allowable differences between both measurements of the section elevations.

The importance comes from the reliability of the preliminary assessment in applied geodesy and the conclusions based on the results [1 – 3]. Other profits are avoiding repeated measurements, and as a result, saving time and money without lack of accuracy in the scientific activities [4 – 8].

Therefore, the allowable discrepancies in the levelling sections in Bulgaria, which are fairly criticized in [9], should be based on scientific analysis [10] and the practice data collections [11].

If the last had been done, [12] might offer more reasonable limits than (1).

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$$|d| \leq 1,5 \text{ mm } \sqrt{l}. \quad (1)$$

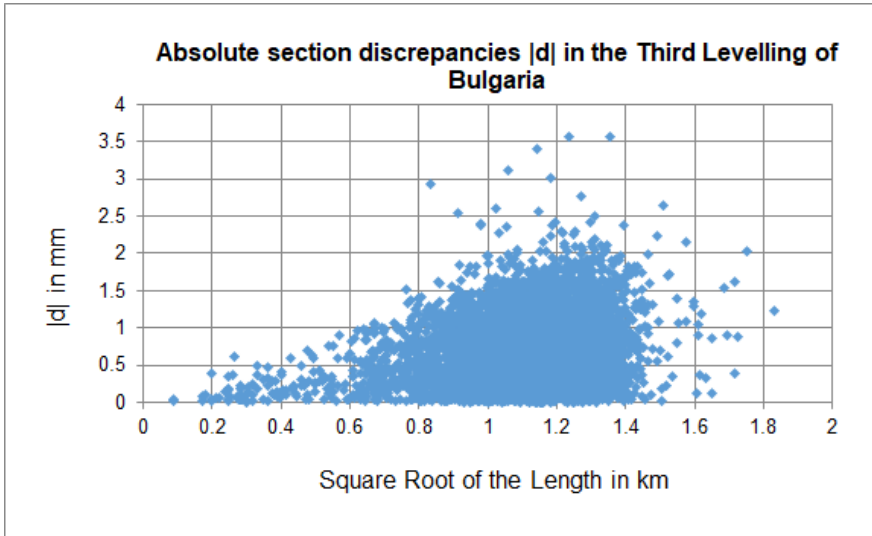


Figure 1. Absolute section discrepancies in the Third Levelling of Bulgaria

Fig. 1 shows the reasons why (1) is not the best choice of allowable limits, i.e.:

- A. There is an obvious presence of heteroscedasticity when the discrepancies are expressed as a function of the square root of the levelling distance. Heteroscedasticity is a violation of the assumptions of the linear regression modeling, and so it can impact the validity of (1). As a result, no correct confidence bands of the predicted values can be produced [13 – 18].
- B. The coefficient of correlation between $|d|$ and $l^{0.5}$ is estimated to be 0,2358. Thus, the square root of the levelling distance explains 5,56% of the variance of the discrepancies. Looking at Fig. 1 one can see that the maximal values of $|d|$ are in the range 0,6 – 2 km and the last ones are approximately twice greater than the discrepancies in these sections whose length is greater than 2 km.

Considering the above facts, it seems that (1) is a rule of thumb rather than a scientifically based argument. Consequently, another decision should be found.

2. Statistical Analysis of the Discrepancies in the Sections

The first statistical analysis of the precise levelling in Bulgaria was made almost 50 years ago [10]. This analysis is based on the data which were obtained in the First and the Second Levelling of Bulgaria. The analysis below refers to the Third Levelling of our country and can be accepted as a sequel of [10].

2.1. Descriptive Statistics

The Third Levelling Net of Bulgaria contains 4573 levelling sections. Their lengths are in the range 0,008 – 3,35 km. The average length is 1,231 km [11]. The sections are commonly

measured twice in the forward and backward directions. Based on both measurements, the discrepancies d are calculated. General descriptive statistics about them are given in Table 1.

Table 1. Descriptive Statistics

Description	Value, mm
Mean	0,178
Standard Error	0,013
Median	0,220
Mode	0,380
Standard Deviation	0,870
Sample Variance	0,756
Minimum	-2,570
Maximum	3,570
Sum	812,446
Count	4573

Analyzing Table 1 it can be seen that the mean of the discrepancies is equal to 0,178 mm. This is a consequence of the fact that 2678 discrepancies are positive and 1895 differences are negative. Also, the maximal positive discrepancy appears twice and is 1 mm greater than the maximal negative one. As a result, the accumulation of positive discrepancies is 812,446 mm in the whole net. This fact simply shows that the absolute values of the upward measurements are greater than the corresponding downward measurements of the elevations in the sections.

In order to investigate whether there is some violation of the normality assumptions, the allowable difference between the numbers of the positive and negative discrepancies is calculated. Since the empirical probability of the appearance of the positive and negative differences is $p = q = 0,5$ and the normal distribution is an approximation of the binomial distribution [19], then the difference between the numbers of the positive and negative discrepancies cannot be greater than some concrete value $2f$, given by (2).

$$f = c^+ - np = t(\alpha, dt)\sqrt{npq}. \quad (2)$$

In equation (2) $n = 4573$ and $c^+ = 2678$. Let $\alpha = 0,05$. Thus, $t = 1,96$ and $f = 66$. According to the analyzed data $c^+ - c^- = 2678 - 1895 = 783 = 2f$ -observed.

Therefore, on 95% confidence level the number of positive discrepancies c^+ is not equal to the number of the negative discrepancies c^- , which fact implies that the discrepancies d do not have normal distribution.

2.2. Pearson's c^2 Test

Graphical visualization of distribution of the analyzed discrepancies is given in Fig. 2 and the results from a c^2 Test of normality [19] are presented in Table 2.

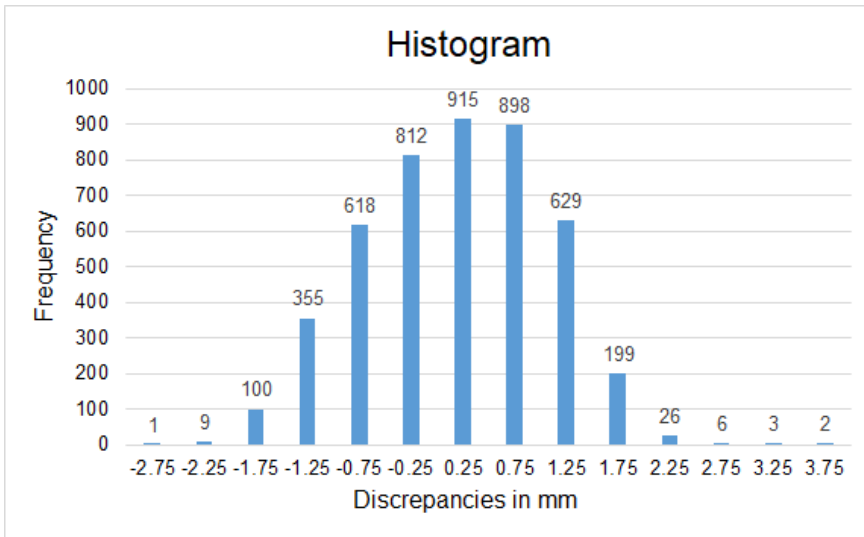


Figure 2. Histogram of the discrepancies

Table 2. c^2 Test of normality

From	To	Obs	PDF _{from}	PDF _{to}	Exp.	(O-E) ² / E
-3,00	-2,50	1	0,00013	0,00104	4,1534	2,3942
-2,50	-2,00	9	0,00104	0,00613	23,3054	8,7810
-2,00	-1,50	100	0,00613	0,02684	94,7058	0,2960
-1,50	-1,00	355	0,02684	0,08781	278,8164	20,8163
-1,00	-0,50	618	0,08781	0,21789	594,8530	0,9007
-0,50	0,00	812	0,21789	0,419054	919,9076	12,6579
0,00	0,50	915	0,41905	0,64457	1031,2823	13,1114
0,50	1,00	898	0,64457	0,82785	838,1553	4,2729
1,00	1,50	629	0,82785	0,93584	493,8081	37,0120
1,50	2,00	199	0,93584	0,98195	210,8686	0,6680
2,00	2,50	26	0,98195	0,99622	65,2501	23,6103
2,50	3,00	6	0,99622	0,99941	14,6262	5,0875
3,00	3,50	3	0,99941	0,99993	2,3741	0,1650
3,50	4,00	2	0,99993	1,00000	0,2789	10,6192
C^2_{obs}						140,392

According to Table 2, all discrepancies are grouped in 14 bins. Thus, the degree of freedom $df = 14 - 3 = 11$ and $c^2 (\alpha = 0,001, 11) = 31,2641 < 140,392$. Therefore, the null hypothesis that the discrepancies in the levelling sections follow a normal distribution have to be rejected on the confidence level of 9,99%. These results are in a full concordance with the results obtained in [10].

2.3. One-Sample Kolmogorov-Smirnov Test of Normality Fit

The second test of normality is the One-Sample Kolmogorov-Smirnov test, which is given in Table 3.

According to Table 3, the maximal difference $D_{\max} = 0,0301 > 0,0288 = D_{crit}$ ($\alpha = 0,001$), where D_{crit} is given by (3) [20].

$$D_{crit} = \frac{1,94947}{\sqrt{n}} = \frac{1,94947}{\sqrt{4573}} = 0,0288. \quad (3)$$

Therefore, the null hypothesis that the distribution of the discrepancies in the levelling sections fits a normal distribution have to be rejected on the confidence level of 99,99%.

Table 3. One-Sample Kolmogorov-Smirnov Test of Normality

To	Cumulative	P_{obs}	P_{exp}	$ P_{obs} - P_{exp} $
-2,50	1	0,0002	0,0010	0,0008
-2,00	10	0,0022	0,0061	0,0039
-1,50	110	0,0241	0,0268	0,0028
-1,00	465	0,1017	0,0878	0,0139
-0,50	1083	0,2368	0,2179	0,0189
0,00	1895	0,4144	0,4191	0,0047
0,50	2810	0,6145	0,6446	0,0301
1,00	3708	0,8108	0,8279	0,0170
1,50	4337	0,9484	0,9358	0,0126
2,00	4536	0,9919	0,9819	0,0100
2,50	4562	0,9976	0,9962	0,0014
3,00	4568	0,9989	0,9994	0,0005
3,50	4571	0,9996	0,9999	0,0004
4,00	4573	1,0000	1,0000	0,0000

2.4. Summary

Based on the above results some general conclusions can be made, i.e.:

- A. The expected value of the discrepancies in the levelling sections is 0,178 mm. The median value is 0,22 mm. Both of them are approximately equal to the station error [12].
- B. Nevertheless, on the length of the sections, the discrepancies in the range -0,50 – 0,50 mm are 1727 out of 4573 or 37,77%. The discrepancies, whose absolute values are less than 1 mm are 3243 out of 4573 or 71%. If the distribution of the discrepancies was normal, we could say that the sample standard is less than 1 mm. But we cannot.
- C. The hypothesis of the normality concerning the section discrepancies in the precise levelling of Bulgaria must be rejected. Consequently, the procedure of determining the allowable limits in levelling sections should be based on the real distribution of the discrepancies.

3. Section Discrepancies Based on Real Data

Table 4 contains probabilities a discrepancy to be less than some given value.

Table 4. Probabilities of $|d|$ appearance

$P(d < x),$ %	$x,$ mm
37,765	0,50
70,916	1,00
92,434	1,50
98,972	2,00
99,738	2,50
99,891	3,00
99,956	3,50
100,000	4,00

Based on Table 4, which is built on a huge volume of real precise levelling data, more appropriate preliminary assessment [2] in applied geodesy can be made.

Also, a reasonable limit of the differences between both measurements in the first order levelling section can be pointed out. It is obvious that 99% of the discrepancies cannot exceed 2 mm despite the distance and measured elevation in the sections.

If one prefers percentiles, then they can use Table 5.

Table 5. Percentiles of $|d|$

Percentile, th	$x,$ mm
50,0	0,67
75,0	1,08
90,0	1,42
95,0	1,60
97,5	1,78
99,0	2,02
99,99	3,02

4. Conclusion

The precise levelling in Bulgaria has evolved through the years. The method is getting more accurate [10 – 11, 21] but there are a lot of tasks to be done [22]. Therefore, the allowable limits of the differences between both measurements of the elevations in the sections, lines and loops should follow this tendency. The results obtained in the previous cycle should be used to define the criterion of the allowable limits, e.g., the results from the Third Levelling should be based on the Fourth Levelling limits, the results from the Fourth Levelling should be based on the Fifth Levelling limits, etc.

Statistics should be the preferable approach of data analyzing rather than old-fashioned methods [23].

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ДОПУСТИМИ РАЗЛИКИ В НИВЕЛАЧНИТЕ СЕКЦИИ, ОСНОВАНИ НА РЕАЛНИ ДАННИ

В. Цветков¹

Ключови думи: прецизна нивелация, грешки, разлики

РЕЗЮМЕ

С цел дефинирането на допустими стойности на разликите от двете измервания на превишенията в нивелачните секции от първокласната нивелачна мрежа на България е извършено изследване на въпросните разлики, получени при третото измерване на мрежата.

Установено е, че изследваните разлики не са нормално разпределени.

Предложени са допуски за разликите между двете измервания в секциите, базирани на реалния закон за разпределението им от последното цялостно измерване на мрежата.

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