ГОДИШНИК НА УНИВЕРСИТЕТА ПО АРХИТЕКТУРА, СТРОИТЕЛСТВО И ГЕОДЕЗИЯ
СОФИЯ

Том Volume 55 | 2022 | Брой Issue 2

ANNUAL OF THE UNIVERSITY OF ARCHITECTURE, CIVIL ENGINEERING AND GEODESY
SOFIA

Получена: 11.02.2022 г.
Приета: 15.04.2022 г.

HOMOGENEITY IN THE THIRD LEVELLING OF BULGARIA

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Keywords: *precise levelling, homogeneity, weights*

ABSTRACT

In order to investigate the homogeneity of the Third Levelling of Bulgaria some nonparametric tests are applied.

A heterogeneity of the net by the Kruskal-Wallis *H* Test has been established.

Analyzing five lines with approximately equal lengths, which pass through different types of relief, the heterogeneity has been detected.

1. Introduction

The heterogeneity of a precise levelling net is an important issue because of the fact that it is connected with the correct choice of the weights which are used in the net adjustment. The understanding of the factors which influence the levelling accuracy is likely to lead to the choice of more appropriate weights. As a result, more accurate conclusions will be produced concerning scientific [1, 7] and applied engineering tasks [8] where the precise levelling is utilized.

Analyzing the First and the Second Levelling of Bulgaria, a lack of normality of the discrepancies was detected [9].

In order to extend the investigation, which was done by [9], the data of the Third Levelling of Bulgaria [10] are examined.

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2. Statistical Analysis of the Homogeneity in the Third Levelling of Bulgaria

In comparison with the First and the Second Levelling of Bulgaria, where the negative discrepancies prevail the positive ones, the Third Levelling reveals the opposite tendency. There are 2678 positive out of 4573 section discrepancies.

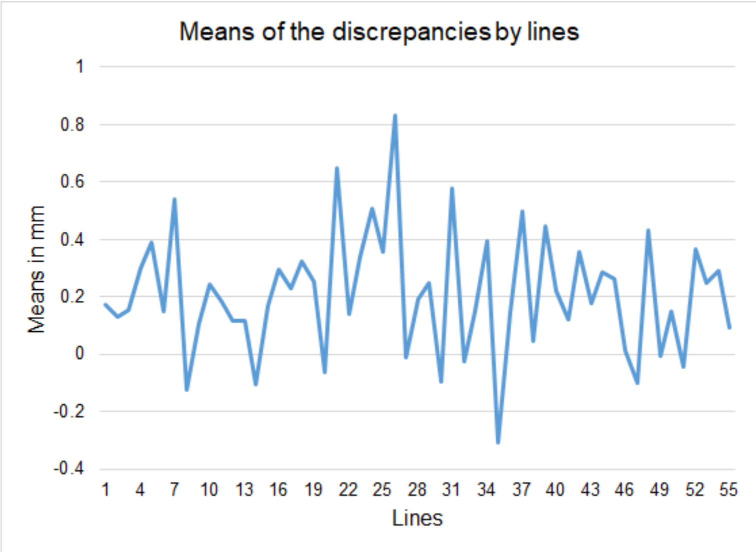

Figure 1. Means of the discrepancies by lines

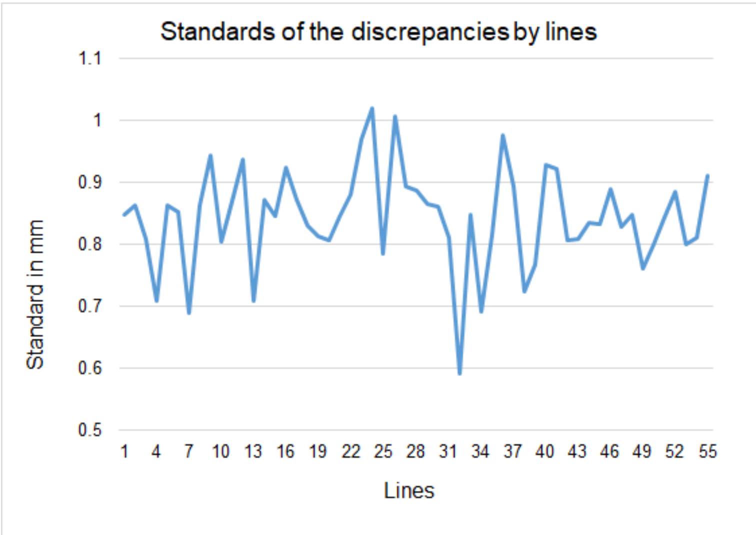

Figure 2. Standards of the discrepancies by lines

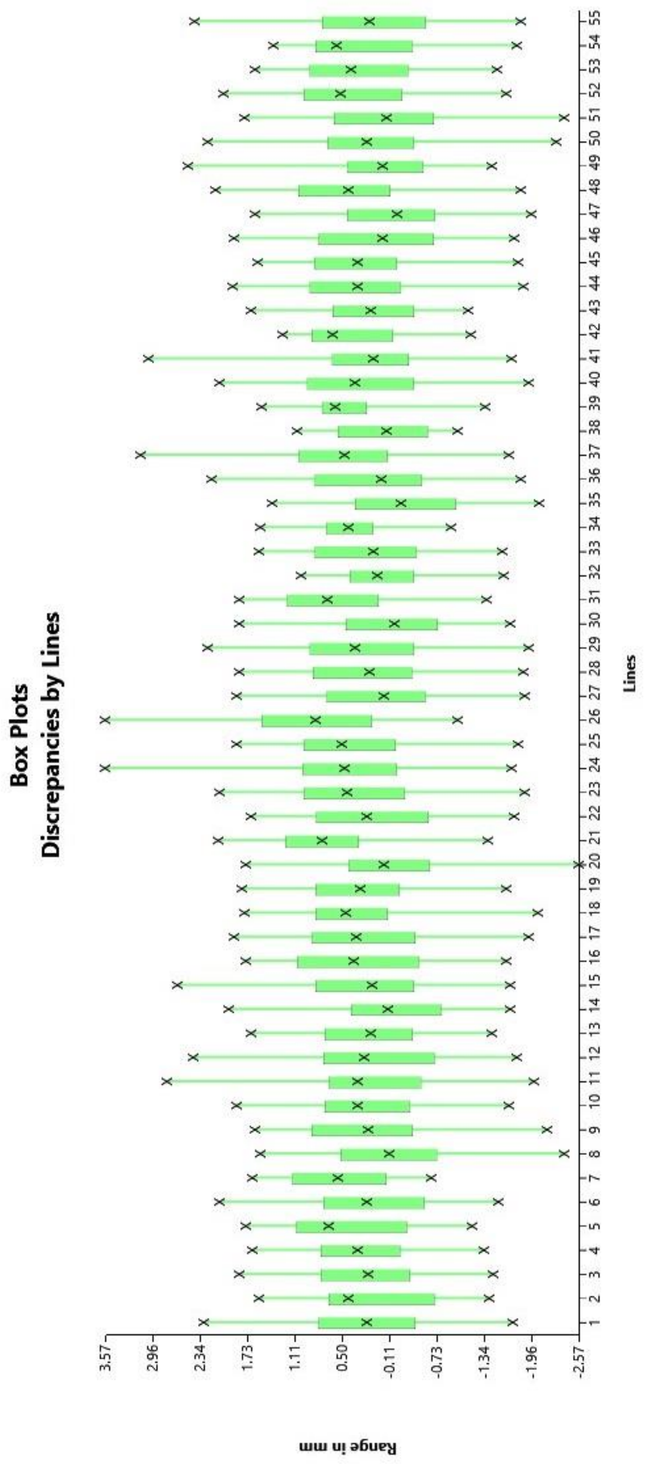

Figure 3. Box Plots

Based on the fact that the empirical probability of the appearance of the positive and negative differences is $p = q = 0{,}5$ and the normal distribution is an approximation of the binomial distribution, then the difference between the numbers of the positive and negative discrepancies cannot be greater than some concrete value $2f$, given by (1).

$$f = c^{+} - np = t(\alpha, dt)\sqrt{npq}. \quad (1)$$

In equation (1) $n = 4573$ and $c^{+} = 2678$. Let $\alpha = 0{,}05$. Thus, $t = 1{,}96$ and $f = 66$. According to the analyzed data $c^{+} - c^{-} = 2678 - 1895 = 783 = 2f$-observed.

Therefore, on 95% confidence level the number of positive discrepancies c^{+} is not equal to the number of the negative discrepancies c^{-}, which fact implies that the discrepancies d do not have normal distribution.

The means of the section discrepancies by lines are pictured in Fig. 1. The standard of these discrepancies is given in Fig. 2.

Looking at Fig. 1 one can count that 10 out of 55 lines have negative means of the discrepancies. It can also be seen that the absolute values of the positive means are commonly greater than the negative ones.

In order to get general view of the distributions of the discrepancies by lines, a Box Plot chart was constructed – Fig. 3. According to Fig. 3, there are obvious differences in the boxes of the separate lines. This fact clearly shows a heterogeneity of the accuracy by lines.

Owing to the violation of the normality in the distribution of the discrepancies, nonparametric tests are only applied below.

2.1. Kruskal-Wallis *H* Test

In order to investigate whether the discrepancies in all lines in the levelling net have the same distribution, the Kruskal-Wallis *H* Test is used. The yielded results are given in Table 1.

Table 1. Kruskal-Wallis *H* Test

Description	Denoted	Value
Groups	n	55
Kruskal-Wallis *H*	H	211,67736
Degree of freedom	$df = n - 1$	54
Alpha	α	0,05
$c^2(df,\alpha)$	c^2_{obs}	72,15322
$P(H,a)$	p-Value	0,00000
$\mathbf{H1: M1 \neq M2 \neq ... \neq Mn}$		

According to the produced p-Value in Table 1, the null hypothesis that all lines have equal medians must be rejected on confidence level greater than 99,99%.

More information about Kruskal-Wallis *H* Test one is able to be found in [9, 11 – 12].

2.2. Test of Homogeneity of Five Lines with an Identical Length

2.2.1. General Data

Five lines which have approximately equal lengths but pass through different types of relief are analyzed in this section. Table 2 contains general data, which describe these lines.

According to Table 2, the elevation between both end bench marks in line 26 is the least of all but the accumulated elevation along the line is the biggest one. Also, the discrepancies |*D*| are the greater. Line 26 passes through the Balkan Mountains. Comparing |*H*| and [|*h*|] in line 26, it is obvious that there is not any compensation of the discrepancies in its sections.

In contrast with line 26 is line 32 which passes through flatter route. Due to this fact there is almost full compensation of the discrepancies in its sections. According to Table 3, the mean of the discrepancies in this line is close to the expected value 0. Line 32 has the least standard of the discrepancies.

Other three lines 2, 10, 18 have identical *L*, |*H*| and [|*h*|] and *s* but different |*D*| and as a result different means m of the discrepancies.

Table 2. General Data

Line	**From**	**To**	**|*D*|, mm**	***L*, km**	**|*H*|, m**	**[|*h*|], m**
2	Lom	Montana	5,68	54,36	111,49	579,10
10	Grivicha	Levski	10,56	45,82	202,93	309,08
18	Dobrich	Varna	14,30	51,73	161,18	663,99
26	Kazanlak	Gabrovo	38,36	50,10	13,15	1654,11
32	Knezha	Zlatna Panega	1,09	52,32	34,34	532,45

Table 3. Discrepancies statistics

Line	**From**	**To**	***m*, mm**	***s*, mm**
2	Lom	Montana	0,132	0,865
10	Grivicha	Levski	0,246	0,804
18	Dobrich	Varna	0,325	0,832
26	Kazanlak	Gabrovo	0,834	1,008
32	Knezha	Zlatna Panega	-0,027	0,593

Using a Two-Sample Kolmogorov-Smirnov Test all combination of pairs between the above-mentioned lines are tested in order to determine whether the lines in each pair derive from equal distribution.

2.2.2. Two-Sample Kolmogorov-Smirnov Test

Table 4 contains the results yielded by ten Two-Sample Kolmogorov-Smirnov Tests [6, 8]. The first column in the table contains the pair of both lines. The second and the third columns give information about the number of the levelling sections in the first and the second line, respectively. The fourth column shows the greatest differences in the frequencies for each pair. The fifth and the last column contain the critical values for $\alpha = 0,05$ and the conclusion whether the null hypothesis H_0 is accepted or rejected H_1, respectively.

The number of the groups is determined according to Sturges' rule [14].

Looking at Table 4 one can see that differences between the distributions of the pair 26 – 2 and 26 – 32 are only detected.

Table 4. Two-Sample Kolmogorov-Smirnov Test

Pairs	*m*	*n*	D_{max}	*D*(0,05,*m*,*n*)	Test
26 – 2	46	43	0,3544	0,288	H_1
26 – 10	46	43	0,2331	0,288	H_0
26 – 18	46	44	0,2332	0,286	H_0
26 – 32	46	41	0,4406	0,292	H_1
32 – 2	41	43	0,1679	0,296	H_0
32 – 10	41	43	0,1849	0,296	H_0
32 – 18	41	44	0,1946	0,295	H_0
2 – 10	43	43	0,1628	0,293	H_0
2 – 18	43	44	0,1432	0,291	H_0
10 – 18	43	44	0,0740	0,291	H_0

2.2.3. Mann-Whitney *U* Test

The two-sample variant of the Kruskal-Wallis Test is a Mann-Whitney *U* Test.

Table 5. Mann-Whitney *U* Test

Pairs	*m*	*n*	*U*	Z_{obs}	Test	
					One-Tail	Two-Tail
26 – 2	46	43	598	3,2104	H_1	H_1
26 – 10	46	43	652	2,7670	H_1	H_1
26 – 18	46	44	714,5	2,4015	H_1	H_1
26 – 32	46	41	451	4,1838	H_1	H_1
32 – 2	41	43	739,5	1,2664	H_0	H_0
32 – 10	41	43	681,5	1,7854	H_1	H_0
32 – 18	41	44	622,5	2,4500	H_1	H_1
2 – 10	43	43	867	0,4923	H_0	H_0
2 – 18	43	44	839	0,9042	H_0	H_0
10 – 18	43	44	890	0,4755	H_0	H_0

Table 5 contains the results yielded by ten Mann-Whitney *H* Tests [11, 15]. The first column in the table contains the pair of both lines. The second and the third columns give information about the number of the levelling sections in the first and the second line, respectively. The fourth and fifth columns show the produced *U* statistic and Z_{obs} value. The last two columns give the conclusions that whether or not the null hypothesis H_0 is accepted depends on the test tails.

According to Table 5, the median of the discrepancies of line 26 differ from the medians of other lines. Also, a difference between lines 32 and 18 is detected.

3. Conclusion

The results of the above analysis reveal a heterogeneity of the precise levelling net in the Third Levelling of Bulgaria. The heterogeneity is due not only to the different length of the lines in the net but also to the accumulation of measured elevations along the lines.

Five lines with approximately equal length can be distinguished from each other and can be classified in three different groups concerning the distributions and medians of their section discrepancies. This fact raises a question of the relevance of the weights which are based on the length of the levelling lines.

In order to clarify the accumulation of the discrepancies along the levelling lines, the data [10] must be digitized in an open file format so that it can be easily accessible [16] by analyzers for further investigations.

Designed experiments on training polygons [16] should be planned, executed and analyzed.

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ХОМОГЕННОСТ НА ТРЕТАТА ПРЕЦИЗНА НИВЕЛАЦИЯ НА БЪЛГАРИЯ

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Ключови думи: *прецизна нивелация, грешки, разлики*

РЕЗЮМЕ

С цел изучаване на хомогенността на първокласната нивелачна мрежа на страната ни, резултатите от третата нивелация на България са анализирани посредством някои непараметрични тестове.

Установена е хетерогенност в точността на отделните линии чрез теста на Крускал-Уолис.

Анализирани са пет нивелачни линии, имащи приблизително еднаква дължина, но преминаващи през различни теренни условия, за които разнородността в точността е подтвърдена.

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