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Weighted Mean Squared Error Criterion with Fixed-Level Modification for Linear-Phase FIR Filter Design*

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Abstract. This paper describes a new approach—a Fixed-Level Least Squares (FLLS) method for linear-phase FIR filter design. It is intended for rejection of the Gibbs phenomenon through the introduction of a set of equally spaced fixed levels in the transition band and subsequent redefinition of the approximated and weighted functions. Detailed mathematical solutions of the problem as well as many examples are given. The results in graphical form are shown as an output of the FLLS software model.

1. Background

The Least Squares (LS) method gained its practical application as an alternative to the well-known McClellan–Parks (MP) approach [8] for the design of linear-phase FIR digital filters. There are numerous publications [see reference list], in which the advantages and the shortcomings of the LS method are discussed, and new modifications of this method are proposed. For example, in [6] examination of the LS method in the time domain is done on the basis of the already-known input autocorrelation function and the crosscorrelation function between the input and the desired output. Vaidyanathan et al. [17] defined a new term *eigenfilter*: a filter, completely constructed according to the LS method, whose coefficients are the components of an eigenvector of a real, symmetric, and positive-definite matrix. The weighted LS approach for the design of filters with equiripple passbands and stopbands is discussed in [2]. Adams et al. [1] examined an extension to the LS method where filters with minimax passband and least-squares stopband are designed. An effective optimizational method for the design of high-order filters with discrete coefficients is given in [7], and some popular and widely used design formulas are mentioned in [5]. Tiajev [15], [16] describes a procedure for the design of LS filters with a flat passband frequency response by the introduction of

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a quadratic trinomial. Very simple from a computational viewpoint is the method of [4], which minimizes the least-square error in the passband only. The application of the LS approach for Type 1 and Type 2 filters is given in detail in [9], [10].

The basic problem with the LS method, as well as with the other approximation methods for FIR filter design, is the Gibbs phenomenon [12], which shows as a "ripple," found near the edge of the passband. It is a result of the discontinuity of the desired frequency response. According to [11], there are four approaches for reducing the overshoot (Gibbs phenomenon) occurring near a discontinuity in the LS method. The first approach involves the introduction of a function that eliminates the discontinuity in the transition band—spline function [3], straight line [11], trigonometric function, etc. In the second approach the error criterion is changed: the transition band is not put under optimization (the so-called don't care region) [4]. The third and fourth approaches use a positive weight function [11] and window functions [11], [12], [14], respectively.

Here a new approach is presented that is a modification of the first and third LS approaches mentioned above. Redefinition of the approximated and the weight functions in the transition band is done with the help of the introduction of f numbers of equally spaced fixed levels in this band (FLLS method).

2. Problem formulation and solution

To solve the problem we use as a starting point the well-known relationship for the least-mean square error:

$$E = \int_0^{0.5} Q(\omega)[A(\omega) - \Phi(\omega, \vec{c})]^2 \, d\omega \tag{1}$$

where $Q(\omega)$ is a positive weight function, $A(\omega)$ is the approximated (given) function, and $\omega \in [0, 0.5]$ is the normalized frequency. $\Phi(\omega, \vec{c})$ serves as an approximation function of the given one $A(\omega)$, and for a Type 1 filter it is

$$\Phi(\omega, \vec{c}) = \sum_{l=0}^{k} c_l \cos l 2\pi \omega \tag{2}$$

where $k = (N-1)/2$, and N is the order of the filter. The coefficients of the digital transfer function

$$H(z) = \sum_{l=0}^{N-1} b_l z^{-l} \tag{3}$$

correspond to the coefficients c_l of $\Phi(\omega, \vec{c})$ as

$$b_{k+l} = b_{k-l} = \frac{c_l}{2}, \qquad l = 1, \ldots, k \tag{4a}$$

$$b_k = c_0. \tag{4b}$$

It is known [5], [11], [17] that the upper defined weight least-mean squared estimation for proximity between the $A(\omega)$ and $\Phi(\omega, \vec{c})$ functions leads to the solution of the set of linear equations

$$\sum_{l=0}^{k} d_{n,l} c_l = d_{n,k+1}, \qquad n = 0, \ldots, k. \tag{5}$$

The coefficients $d_{n,l}$ and the free term $d_{n,k+1}$ are defined by the relationships

$$d_{n,l} = \int_0^{0.5} Q(\omega) \cos(2\pi n \omega) \cos(2\pi l \omega)\, d\omega \tag{6a}$$

$$d_{n,k+1} = \int_0^{0.5} Q(\omega) A(\omega) \cos(2\pi n \omega)\, d\omega. \tag{6b}$$

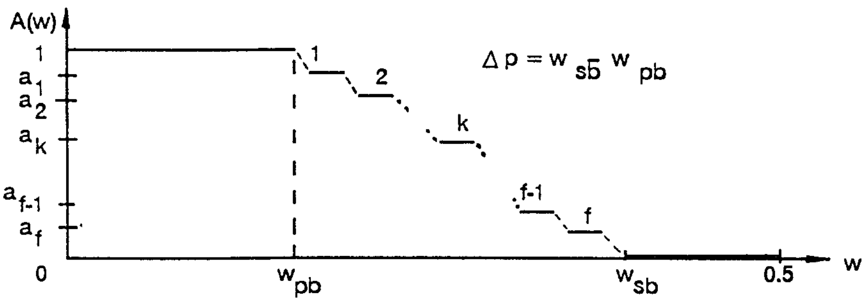

Figure 1.

The most important aspect of the presented FLLS method is the redefinition of the approximated function $A(\omega)$ in the transition band and the corresponding changes in the weighted function $Q(\omega)$. Thus, for example, for a lowpass filter f ($f \geq 1$) fixed levels in the transition band are set, with the intention of eliminating the Gibbs phenomenon (Figure 1). Here ω_{pb} (passband) and ω_{sb} (stopband) are normalized frequencies in the passband and the stopband of the filter, respectively. The transition band Δp is divided into $(2f + 1)$ equally spaced sublevels. The values of $A(\omega)$ and $Q(\omega)$ for the sublevels, shown in Figure 1 with a solid line, are given in Table 1. In the zones drawn with a dashed line in Figure 1, the approximated function $A(\omega)$ remains undefined and the weight function $Q(\omega)$ has a zero value. The vector $\vec{a} = (a_1, a_2, \ldots, a_f)$ describes the fixed levels of $A(\omega)$ in the transition band. It has been set that the values of a_i, $i = 1, 2, \ldots, f$, divide the [0, 1] interval on the $A(\omega)$ axis into $i + 1$ equal parts. In other words, the elements of the vector $\vec{a}$ are defined for a lowpass filter according to the relationship

$$a_i = \frac{f - i + 1}{f + 1}, \qquad i = 1, 2, \ldots, f.$$

For example if we have three levels in the transition band ($f = 3$) the vector $\vec{a}$ is $\vec{a} = \left[\frac{3}{4}, \frac{2}{4}, \frac{1}{4}\right]$. The elements of the vector $\vec{t} = (t_1, t_2, \ldots, t_f)$ from Table 1

Table 1.

Lowpass filter (LP) $\Delta p = \omega_{sb} - \omega_{pb}$ $\omega_1 = \omega_{pb}$; $\omega_2 = \omega_{sb}$	Highpass filter (HP) $\Delta p = \omega_{pb} - \omega_{sb}$ $\omega_1 = \omega_{sb}$; $\omega_2 = \omega_{pb}$	$A(\omega)$	$Q(\omega)$
$\omega \in [0, \omega_1]$		1 for LP filter 0 for HP filter	g_1
$\omega \in \left[\omega_1 + \frac{1}{2f+1}\Delta p, \omega_1 + \frac{2}{2f+1}\Delta p\right]$		a_1	t_1
$\omega \in \left[\omega_1 + \frac{3}{2f+1}\Delta p, \omega_1 + \frac{4}{2f+1}\Delta p\right]$		a_2	t_2
....		......	......
$\omega \in \left[\omega_1 + \frac{2f-3}{2f+1}\Delta p, \omega_1 + \frac{2f-2}{2f+1}\Delta p\right]$		a_{f-1}	t_{f-1}
$\omega \in \left[\omega_1 + \frac{2f-1}{2f+1}\Delta p, \omega_1 + \frac{2f}{2f+1}\Delta p\right]$		a_f	t_f
$\omega \in [\omega_2, 0.5]$		0 for LP filter 1 for HP filter	g_2

are equal to the values of $Q(\omega)$ for the given fixed levels. The positive weight coefficients g_1, g_2 refer to the passband, stopband (LP filter) and to the stopband, passband (HP filter).

The approach for the derivation of $d_{n,l}$ and $d_{n,k+1}$ for an LP filter in the FLLS method is given below. From the formulas (6a) and (6b) it is clear that to determine $d_{n,l}$ one must know the values of $Q(\omega)$ only, and to determine $d_{n,k+1}$ one must know the values of $A(\omega)$ and $Q(\omega)$. The values of $d_{n,l}$ and $d_{n,k+1}$ have been calculated for $f = 1, \ldots, 6$ consecutively. The observed tendencies were summarized for arbitrary f. In the process of determination of $d_{n,l}$, three different cases have been examined according to the correlation between n and l; $n = l = 0$, $n = l \neq 0$, $n \neq l$. By analogy with these three cases, two possible cases have been taken into account for $d_{n,k+1}$: $n = 0$, $n \neq 0$. For an LP filter the final results were derived as follows (here the coefficients $d^*_{n,l}$ and $d^*_{n,k+1}$ [9], which refer to the "standard" LS method, are given in the Appendix):

(a) $d_{n,l}$, $n = 0, 1, \ldots k$; $l = 0, 1, \ldots k$

(i) for $n = l = 0$:

$$d_{n,l} = d^*_{n,l} + \frac{\Delta p}{2f+1} \sum_{i=1}^{f} t_i.$$

(ii) for $n = l \neq 0$:

$$d_{n,l} = d^*_{n,l} + \frac{\Delta p}{2(2f+1)} \sum_{i=1}^{f} t_i$$

$$+ \frac{\sum_{i=1}^{f} \left\{ t_i \left[\sin 2\pi(n+l) \left(\omega_{pb} + \frac{2i}{2f+1}\Delta p \right) - \sin 2\pi(n+l) \left(\omega_{pb} + \frac{2i-1}{2f+1}\Delta p \right) \right] \right\}}{4\pi(n+l)}$$

$$= d^*_{n,l} + \frac{\Delta p}{2(2f+1)} \sum_{i=1}^{f} t_i + \frac{\sin \frac{\pi(n+l)\Delta p}{2f+1}}{2\pi(n+l)}$$

$$\sum_{i=1}^{f} t_i \cdot \cos \pi(n+l) \left(2\omega_{pb} + \frac{4i-1}{2f+1}\Delta p \right).$$

(iii) for $n \neq l$:

$$d_{n,l} = d^*_{n,l} + \frac{\sin \frac{\pi(n+l)\Delta p}{2f+1}}{2\pi(n+l)} \sum_{i=1}^{f} t_i \cos \pi(n+l) \left(2\omega_{pb} + \frac{4i-1}{2f+1}\Delta p \right)$$

$$+ \frac{\sin \frac{\pi(n-l)\Delta p}{2f+1}}{2\pi(n-l)} \sum_{i=1}^{f} t_i \cos \pi(n-l) \left(2\omega_{pb} + \frac{4i-1}{2f+1}\Delta p \right).$$

(b) $d_{n,k+1}$, $n = 0, 1, \ldots, k$

(i) for $n = 0$:

$$d_{n,k+1} = d^*_{n,k+1} + \frac{\Delta p}{2f+1} \sum_{i=1}^{f} t_i a_i.$$

(ii) for $n \neq 0$:

$$d_{n,k+1} = d^*_{n,k+1}$$

$$+ \frac{\sum_{i=1}^{f} \left\{ t_i a_i \left[\sin 2\pi n \left(\omega_{pb} + \frac{2i}{2f+1}\Delta p \right) - \sin 2\pi n \left(\omega_{pb} + \frac{2i-1}{2f+1}\Delta p \right) \right] \right\}}{2\pi n}$$

$$= d^*_{n,k+1} + \frac{\sin \frac{\pi n \Delta p}{2f+1}}{\pi n} \sum_{i=1}^{f} t_i a_i \cos \pi n \left(2\omega_{pb} + \frac{4i-1}{2f+1}\Delta p \right).$$

The approach mentioned above has been applied to the derivation of $d_{n,l}$ and $d_{n,k+1}$ for HP, BP, and BS filters. The relationships differ from these for an LP filter because $A(\omega)$ and $Q(\omega)$ have different descriptions (for the HP filter, see Table 1; for BP and BS filters, see Table 2). In Table 2 ω_{pb1}, ω_{pb2} (ω_{sb1}, ω_{sb2}) denote the pairs of normalized cutoff frequencies at the passband (stopband), and Δp_1, Δp_2 are the two transition bands for BP and BS filters. For the BP filter g_1 and g_3 are weight coefficients in the stopbands (g_2 in the passband). For the BS filter g_1 and g_3 are weight coefficients in the passbands (g_2 in the stopband).

For BP and BS filters (Table 2) two vectors $\vec{t}'$, $\vec{t}''$ of weight coefficients were introduced as well as vectors $\vec{a}'$, $\vec{a}''$, describing values of the fixed levels in the two transition bands. Here it is assumed that the BP and BS filters have equal numbers of fixed levels f in the transition bands. The values that take the vectors' elements $\vec{a}, \vec{a}', \vec{a}''$ are summarized in Table 3; where $i = 1, 2, \ldots, f$. It is obvious that for BP and BS filters we have $a'_i = a''_{f-i+1}$.

Table 2.

Bandpass filter (BP) | Bandstop filter (BS)	$A(\omega)$	$Q(\omega)$
BP: $\Delta p_1 = \omega_{pb1} - \omega_{sb1}$; BS: $\Delta p_1 = \omega_{sb1} - \omega_{pb1}$ BP: $\Delta p_2 = \omega_{sb2} - \omega_{pb2}$; BS: $\Delta p_2 = \omega_{pb2} - \omega_{sb2}$ BP: $\omega_1 = \omega_{sb1}$; $\omega_2 = \omega_{pb1}$; BS: $\omega_1 = \omega_{pb1}$; $\omega_2 = \omega_{sb1}$ BP: $\omega_3 = \omega_{pb2}$; $\omega_4 = \omega_{sb2}$; BS: $\omega_3 = \omega_{sb2}$; $\omega_4 = \omega_{pb2}$		
$\omega \in [0, \omega_1]$	1 for BP filter 0 for BS filter	g_1
$\omega \in \left[\omega_1 + \frac{1}{2f+1}\Delta p_1, \omega_1 + \frac{2}{2f+1}\Delta p_1\right]$	a'_1	t'_1
$\omega \in \left[\omega_1 + \frac{3}{2f+1}\Delta p_1, \omega_1 + \frac{4}{2f+1}\Delta p_1\right]$	a'_2	t'_2
....	......	......
$\omega \in \left[\omega_1 + \frac{2f-1}{2f+1}\Delta p_1, \omega_1 + \frac{2f}{2f+1}\Delta p_1\right]$	a'_f	t'_f
$\omega \in [\omega_2, \omega_3]$	0 for BP filter 1 for BS filter	g_2
$\omega \in \left[\omega_3 + \frac{1}{2f+1}\Delta p_2, \omega_3 + \frac{2}{2f+1}\Delta p_2\right]$	a''_1	t''_1
$\omega \in \left[\omega_3 + \frac{3}{2f+1}\Delta p_2, \omega_3 + \frac{4}{2f+1}\Delta p_2\right]$	a''_2	t''_2
....	......	......
$\omega \in \left[\omega_3 + \frac{2f-1}{2f+1}\Delta p_2, \omega_3 + \frac{2f}{2f+1}\Delta p_2\right]$	a''_f	t''_f
$\omega \in [\omega_4, 0.5]$	0 for BP filter 1 for BS filter	g_3

After completion of the calculations in accordance with the specific description of $A(\omega)$ and $Q(\omega)$ the final results for all types of filters are given in Tables 4a and 4b.

To clarify the above given formulas, we have introduced additional functions of two, three, or four varibles as follows:

$$S_{1,2}(\Delta, \omega, \vec{x}) = \frac{\sin \frac{\pi(n\pm l)\Delta}{2f+1}}{2\pi(n \pm l)} \sum_{i=1}^{f} x_i \cos \pi(n \pm l)\left(2\omega + \frac{4i-1}{2f+1}\Delta\right),$$

Table 3.

Elements	a_i	a'_i	a''_i
Type of Filter			
Lowpass filter (LP)	$\frac{f-i+1}{f+1}$	$\cdots$	$\cdots$
Highpass filter (HP)	$\frac{i}{f+1}$	$\cdots$	$\cdots$
Bandpass (BP) filter	$\cdots$	$\frac{i}{f+1}$	$\frac{f-i+1}{f+1}$
Bandstop filter (BS)	$\cdots$	$\frac{f-i+1}{f+1}$	$\frac{i}{f+1}$

Table 4a.

Coeff.	$d_{n,l}$	$d_{n,l}$	
Type	$n=l=0$	$n=l\neq 0$	$n\neq l$
LP filter	$d^*_{n,l}+S_4(\Delta p,\vec{t})$	$d^*_{n,l}+0.5S_4(\Delta p,\vec{t})+S_1(\Delta p,\omega_{pb},\vec{t})$	$d^*_{n,l}+S_1(\Delta p,\omega_{pb},\vec{t})+S_2(\Delta p,\omega_{pb},\vec{t})$
HP filter	$d^*_{n,l}+S_4(\Delta p,\vec{t})$	$d^*_{n,l}+0.5S_4(\Delta p,\vec{t})+S_1(\Delta p,\omega_{sb},\vec{t})$	$d^*_{n,l}+S_1(\Delta p,\omega_{sb},\vec{t})+S_2(\Delta p,\omega_{sb},\vec{t})$
BP filter	$d^*_{n,l}+S_4(\Delta p_1,\vec{t}')$ $+S_4(\Delta p_2,\vec{t}'')$	$d^*_{n,l}+0.5S_4(\Delta p_1,\vec{t}')+0.5S_4(\Delta p_2,\vec{t}'')$ $+S_1(\Delta p_1,\omega_{sb1},\vec{t}')+S_1(\Delta p_2,\omega_{pb2},\vec{t}'')$	$d^*_{n,l}+S_1(\Delta p_1,\omega_{sb1},\vec{t}')+S_1(\Delta p_2,\omega_{pb2},\vec{t}'')$ $+S_2(\Delta p_1,\omega_{sb1},\vec{t}')+S_2(\Delta p_2,\omega_{pb2},\vec{t}'')$
BS filter	$d^*_{n,l}+S_4(\Delta p_1,\vec{t}')$ $+S_4(\Delta p_2,\vec{t}'')$	$d^*_{n,l}+0.5S_4(\Delta p_1,\vec{t}')+0.5S_4(\Delta p_2,\vec{t}'')$ $+S_1(\Delta p_1,\omega_{pb1},\vec{t}')+S_1(\Delta p_2,\omega_{sb2},\vec{t}'')$	$d^*_{n,l}+S_1(\Delta p_1,\omega_{pb1},\vec{t}')+S_1(\Delta p_2,\omega_{sb2},\vec{t}'')$ $+S_2(\Delta p_1,\omega_{pb1},\vec{t}')+S_2(\Delta p_2,\omega_{sb2},\vec{t}'')$

$$S_3(\Delta,\omega,\vec{x},\vec{y})=\frac{\sin\frac{\pi n\Delta}{2f+1}}{\pi n}\sum_{i=1}^{f}x_i y_i\cos\pi n\left(2\omega+\frac{4i-1}{2f+1}\Delta\right),$$

Table 4b.

Coeff.	$d_{n,k+1}$	$d_{n,k+1}$
Type	$n=0$	$n \neq 0$
LP filter	$d^*_{n,k+1} + S_5(\Delta p, \vec{t}, \vec{a})$	$d^*_{n,k+1} + S_3(\Delta p, \omega_{pb}, \vec{t}, \vec{a})$
HP filter	$d^*_{n,k+1} + S_5(\Delta p, \vec{t}, \vec{a})$	$d^*_{n,k+1} + S_3(\Delta p, \omega_{sb}, \vec{t}, \vec{a})$
BP filter	$d^*_{n,k+1} + S_5(\Delta p_1, \vec{t}', \vec{a}')$ $+S_5(\Delta p_2, \vec{t}'', \vec{a}'')$	$d^*_{n,k+1} + S_3(\Delta p_1, \omega_{sb1}, \vec{t}', \vec{a}')$ $+S_3(\Delta p_2, \omega_{pb2}, \vec{t}'', \vec{a}'')$
BS filter	$d^*_{n,k+1} + S_5(\Delta p_1, \vec{t}', \vec{a}')$ $+S_5(\Delta p_2, \vec{t}'', \vec{a}'')$	$d^*_{n,k+1} + S_3(\Delta p_1, \omega_{pb1}, \vec{t}', \vec{a}')$ $+S_3(\Delta p_2, \omega_{sb2}, \vec{t}'', \vec{a}'')$

$$S_4(\Delta, \vec{x}) = \frac{\Delta}{2f+1}\sum_{i=1}^{f} x_i, \qquad S_5(\Delta, \vec{x}, \vec{y}) = \frac{\Delta}{2f+1}\sum_{i=1}^{f} x_i y_i,$$

where

$$\vec{x} = (x_1, x_2, \ldots, x_f), \qquad \vec{y} = (y_1, y_2, \ldots, y_f)$$

$$\Delta \in \{\Delta p, \Delta p_1, \Delta p_2\}, \qquad \omega \in \{\omega_{pb}, \omega_{sb}, \omega_{pb1}, \omega_{pb2}, \omega_{sb1}, \omega_{sb2}\}$$

$$\vec{x} = \{\vec{t}, \vec{t}', \vec{t}''\}, \qquad \vec{y} \in \{\vec{a}, \vec{a},' \vec{a}''\}.$$

3. Numerical examples

To illustrate the use of the FLLS method, four complete examples are given.

Example 1. Design of 37-point bandpass filter with stopband cut-off frequencies of 0.1 and 0.38 and passband cut-off frequencies of 0.25 and 0.35 and with ripple weights of 10 in the stopbands and 100 in the passband as follows:

$$N = 37, \quad \omega_{sb1} = 0.1, \quad \omega_{pb1} = 0.25, \quad \omega_{pb2} = 0.35, \quad \omega_{sb2} = 0.38,$$

$$g_1 = g_3 = 10, \quad g_2 = 100.$$

The resulting frequency responses in dB are given in Figures 2a,b as a result of applying the LS and MP approaches, respectively. The 30 dB ripple in the wide transition band is clearly visible. For rejection of this ripple the new FLLS method was applied with:

$f = 1, \qquad \vec{t}' = (100)$ (Figure 2c)

$f = 2, \quad \vec{t}' = (100, 100)$ (Figure 2d)

$f = 3, \quad \vec{t}' = (10, 1, 40000)$ (Figure 2e)

$f = 5, \quad \vec{t}' = (0.05, 10, 1, 1000, 30000)$ (Figure 2f)

In each figure the transition band is given at the upper right corner in larger scale. The resulting responses show significant rejection of the ripple, especially in the case of 3 and 5 fixed levels in the transition band and for larger weight coefficients. But some decrease in attenuation of the left-hand stopband is observed with FLLS.

Example 2. Design of bandstop filter with:

$$N = 59, \quad \omega_{pb1} = 0.03, \quad \omega_{sb1} = 0.2, \quad \omega_{sb2} = 0.3, \quad \omega_{pb2} = 0.43,$$

$$g_1 = 100, \quad g_2 = 1, \quad g_3 = 10.$$

In Figure 3a is given the resulting frequency response, according to the "standard" LS method. The new FLLS method was applied with:

$f = 2, \quad \vec{t}' = (10, 1), \quad \vec{t}'' = (1, 10)$ (Figure 3b)

$f = 3, \quad \vec{t}' = (1, 100, 1), \quad \vec{t}'' = (1, 100, 1)$ (Figure 3c)

$f = 4, \quad \vec{t}' = (150, 140, 1000, 50), \quad \vec{t}'' = (50, 1000, 140, 150)$ (Figure 3d).

The log and linear magnitude characteristics are given. From the linear characteristics (on the right) the stepwise transition bands are clearly visible. After FLLS application the right-hand transition band in Figure 3d becomes more flat (as a result of the small Δp_2 and the suitable chosen values of $\vec{t}''$). Analogously to Example 1, a decrease in attentuation of the stopband is observed.

Example 3. Design of lowpass filter with:

$$N = 25, \quad \omega_{pb} = 0.3, \quad \omega_{sb} = 0.35, \quad g_1 = 10, \quad g_2 = 1000.$$

The resulting characteristics are given in Figure 4:

FLLS method: $f = 2, \vec{t} = (100, \ 18)$, (Figure 4a)

FLLS method: $f = 3, \vec{t} = (10000, \ 10000, \ 100000)$, (Figure 4b)

MP approach (Figure 4c).

The frequency responses, derived with the same input data ($N = 25$, $\omega_{pb} = 0.3$) with rectangular, Hamming, and triangular windows are given in [14, pp. 158–159]. It can be seen that the response of Figure 4b gives greater stopband attenuation (50 dB) than rectangular (20 dB) and triangular (25 dB) windows. The FLLS method here gives attenuation similar to that achieved by the Hamming window [14] and MP approaches.

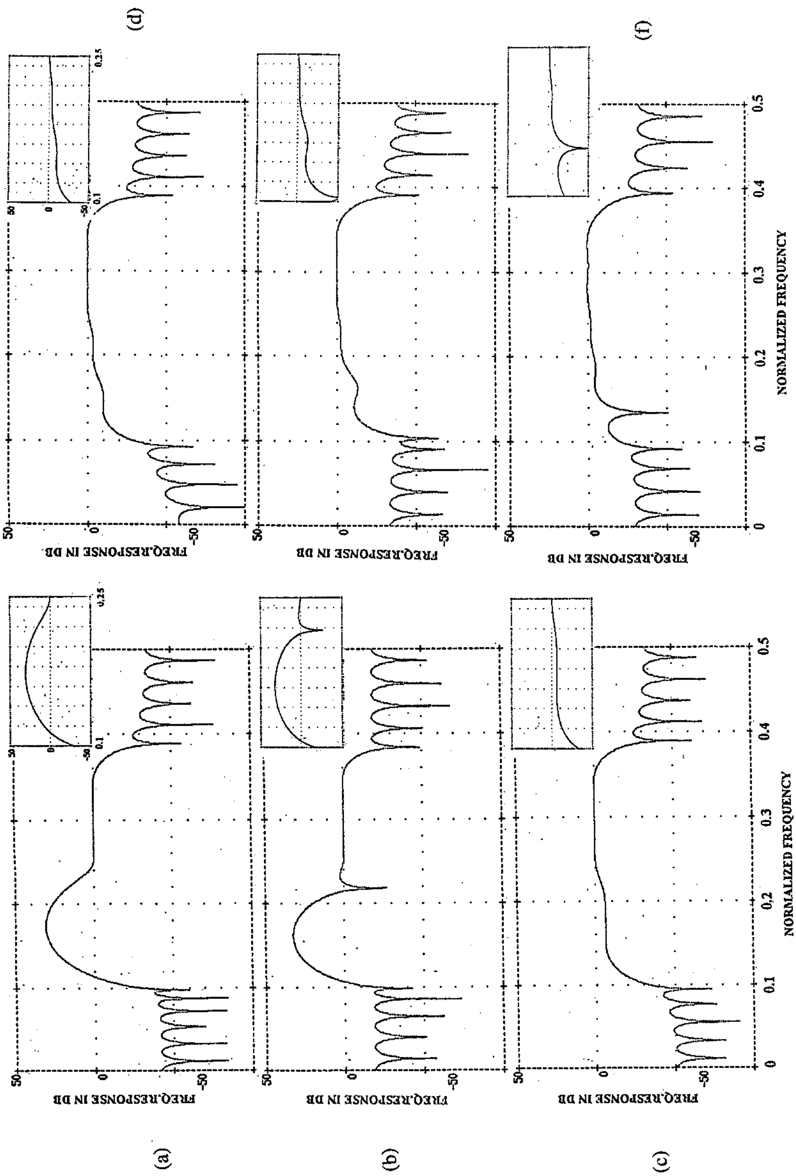
(a)
(b)
(c)
(d)
(f)
FREQ.RESPONSE IN DB
NORMALIZED FREQUENCY
50
0
-50
0
0.1
0.2
0.3
0.4
0.5
0.25

Figure 2.

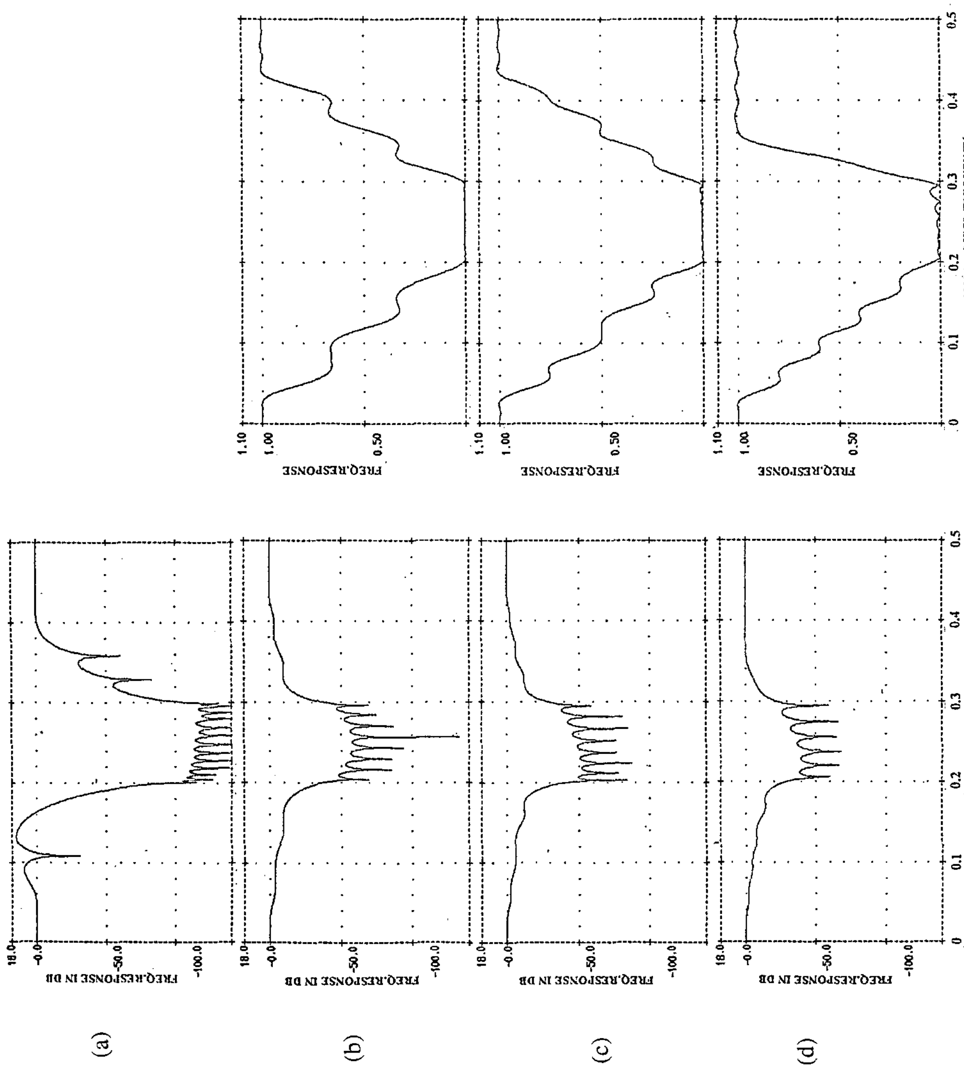
(a)
(b)
(c)
(d)
FREQ.RESPONSE IN DB
FREQ.RESPONSE
NORMALIZED FREQUENCY

Figure 3.

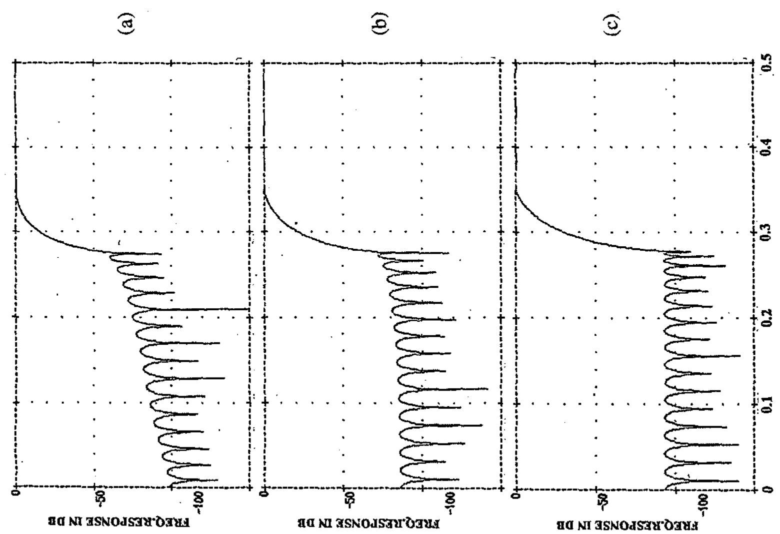

Figure 5.

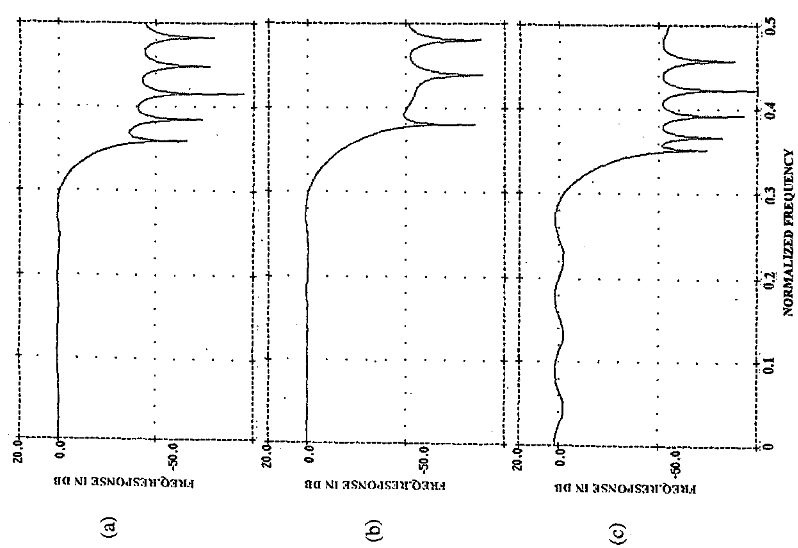

Figure 4.

Table A-1.

Coeff.	$d^*_{n,l}$	$d^*_{n,l}$	$d^*_{n,l}$	$d^*_{n,k+1}$	$d^*_{n,k+1}$
Type	$n = l = 0$	$n = l \neq 0$	$n \neq l$	$n = 0$	$n \neq 0$
LP filter	$g_1\omega_{pb}$ $+g_2(0.5-\omega_{sb})$	$R_1(g_1,\omega_{pb}) - R_1(g_2,\omega_{sb})$ $+0.5g_1\omega_{pb} + 0.5g_2(0.5-\omega_{sb})$	$R_1(g_1,\omega_{pb}) - R_1(g_2,\omega_{sb})$ $+R_2(g_1,\omega_{pb}) - R_2(g_2,\omega_{sb})$	$g_1\omega_{pb}$	$R_3(g_1,\omega_{pb})$
HP filter	$g_1\omega_{sb}$ $+g_2(0.5-\omega_{pb})$	$R_1(g_1,\omega_{sb}) - R_1(g_2,\omega_{pb})$ $+0.5g_1\omega_{sb} + 0.5g_2(0.5-\omega_{pb})$	$R_1(g_1,\omega_{sb}) - R_1(g_2,\omega_{pb})$ $+R_2(g_1,\omega_{sb}) - R_2(g_2,\omega_{pb})$	$g_2(0.5-\omega_{pb})$	$-R_3(g_2,\omega_{pb})$
BP filter	$g_1\omega_{sb1}$ $g_2(\omega_{pb2}-\omega_{pb1})$ $+g_3(0.5-\omega_{sb2})$	$R_1(g_1,\omega_{sb1}) - R_1(g_3,\omega_{sb2})$ $+R_1(g_2,\omega_{pb2}) - R_1(g_2,\omega_{pb1})$ $+0.5g_1\omega_{sb1} + 0.5g_2(\omega_{pb2}-\omega_{pb1})$ $+0.5g_3(0.5-\omega_{sb2})$	$R_1(g_1,\omega_{sb1}) - R_1(g_3,\omega_{sb2})$ $+R_2(g_1,\omega_{sb1}) - R_2(g_3,\omega_{sb2})$ $+R_1(g_2,\omega_{pb2}) - R_1(g_2,\omega_{pb1})$ $+R_2(g_2,\omega_{pb2}) - R_2(g_2,\omega_{pb1})$	$g_2\omega_{pb2}$ $-g_2\omega_{pb1}$	$R_3(g_2,\omega_{pb2})$ $-R_3(g_2,\omega_{pb1})$
BS filter	$g_1\omega_{pb1}$ $+g_2(\omega_{sb2}-\omega_{sb1})$ $+g_3(0.5-\omega_{pb2})$	$R_1(g_1,\omega_{pb1}) - R_1(g_3,\omega_{pb2})$ $+R_1(g_2,\omega_{sb2}) - R_1(g_2,\omega_{sb1})$ $+0.5g_1\omega_{pb1} + 0.5g_2(\omega_{sb2}-\omega_{sb1})$ $+0.5g_3(0.5-\omega_{pb2})$	$R_1(g_1,\omega_{pb1}) - R_1(g_3,\omega_{pb2})$ $+R_2(g_1,\omega_{pb1}) - R_2(g_3,\omega_{pb2})$ $+R_1(g_2,\omega_{sb2}) - R_1(g_2,\omega_{sb1})$ $+R_2(g_2,\omega_{sb2}) - R_2(g_2,\omega_{sb1})$	$g_1\omega_{pb1}$ $+g_3(0.5-\omega_{pb2})$	$R_3(g_1,\omega_{pb1})$ $-R_3(g_3,\omega_{pb2})$

Example 4. Design of highpass filter with:

$$N = 45, \quad \omega_{sb} = 0.27767, \quad \omega_{pb} = 0.35, \quad g_1 = 2000, \quad g_2 = 1.$$

The resulting characteristics are given in Figure 5:

FLLS method: $f = 1, \vec{t} = (0.5)$, (Figure 5a)

LS method (Figure 5b)

MP aproach (Figure 5c).

The Hanning window frequency response for the same input data is given in [12, p. 104]. The stopband attenuation for FLLS (65 dB) is greater than Hanning's (45 dB). The LS and MP approaches have greater attenuation than the FLLS one.

4. Conclusions

This paper offers a new (FLLS) method for application of the Least Squares approach for the design of Type 1 linear-phase FIR filters. For rejection of the Gibbs phenomenon, f numbers of fixed levels are introduced in the transition band. The $A(\omega)$ and $Q(\omega)$ functions are redefined with the aid of a set of vectors. The FLLS mathematical background is given in detail as well as the final relationships for the four types of filters which are summarized in tables. The numerical examples and comparison with other well-known approaches show the merit of this method.

Appendix

The "standard" LS method [9] uses as a basis the formulas (1)–(6), given in Section 2. The weight function $Q(\omega)$ is described with the coefficients g_1, g_2 (for LP and HP filters) and with the coefficients g_1, g_2, g_3 (for BP and BS filters). Solving the set of equations (5), we find the final relationships for the coefficients $d^*_{n,l}$ and $d^*_{n,k+1}$ (see Table A-1). Three additional functions, introduced for describing the final result, are:

$$R_{1,2}(g,\omega) = \frac{g}{4\pi(n \pm l)} \sin 2\pi\omega(n \pm l), \qquad R_3(g,\omega) = \frac{g}{2\pi n} \sin 2\pi\omega n,$$

where

$$g \in \{g_1, g_2, g_3\}, \qquad \omega \in \{\omega_{pb}, \omega_{sb}, \omega_{pb1}, \omega_{pb2}, \omega_{sb1}, \omega_{sb2}\}.$$

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