ANALYTICAL AND FEA DESIGN OF STEEL DOMES WITH RADIAL GIRDERS AND CIRCULAR ELEMENTS

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Abstract: *Domes on circular base are widespread – silos, tanks, bulk storage, sport facilities, exhibition halls. They are light, nice and could cover big spans without internal columns. Steel domes successfully cover spans with diameter* $\boldsymbol{D} \leq 40{,}0$ *m, but aluminium domes could cover spans with diameter* $\boldsymbol{D} \leq 120{,}0$ *m*

Bigger part of domes on circular base has radial girders, circular elements - rings and cover plates. Usually they are steel made.

The domes are spatial statically undetermined systems, so it is difficult to obtain the design stresses and forces without finite element analysis (FEA) model. Accurate design of the elements of the roof structure can be achieved only by using adequate program product and taking into account the rigidity of the elements and flexibility of the joints.

Key words: *self-supporting dome roof, radial girders, rings, roof cover plates, effective length, bending, axial forces, FEA*

1. Introduction

The domes on the circular-shaped base have a large usage – silos, tanks, warehouses for bulk materials, sportive facilities, and exhibition halls. They are light, beautiful and can cover big spans, providing free space without intermediate columns. The steel domes are successfully applied on the spans with diameter $\boldsymbol{D} \leq 40{,}0$ m, aluminium alloys domes can be used to cover spans with diameter $\boldsymbol{D} \leq 120{,}0$ m.

A big part of the domes constructed on the circular base has a radial girders and circular elements, and they are steel made. Traditionally their construction is composed by radial elements (girders), ring-shaped elements (rings) and roof cover plates (Fig. 1) .

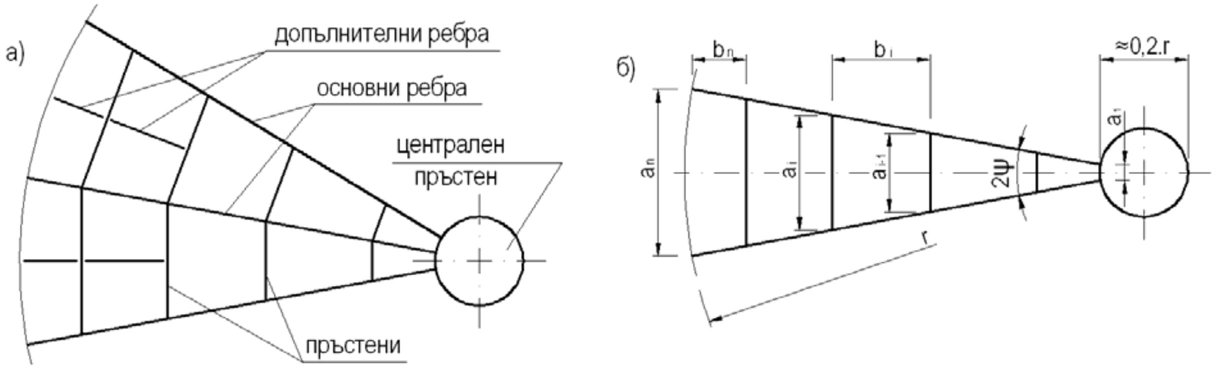

Fig. 1 Structure of the roof
а) basic elements б) design scheme

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2. Loads and load combinations

The calculation of roof domes during the exploitation is done for two load combinations. In the first of them the summarized loading q_1 operates from the top to the bottom. In the second one the loading on the roof q_2 operates from the bottom to the top:

$$q_1 = \gamma_{\mathrm{fg}}.g_n + \gamma_{\mathrm{ft}}.g_{tn} + \psi_{\mathrm{c}}.(\gamma_{\mathrm{fv}}.p_v^n + \gamma_{\mathrm{fs}}.s_n) \downarrow , \tag{2.1}$$

$$q_2 = \psi_{\mathrm{c}}.(\gamma_{\mathrm{fa}}.p_o^n + \gamma_{\mathrm{fw}}.w_n) - \gamma'_{\mathrm{fg}}.(g_n + g_{tn}) \uparrow , \tag{2.2}$$

where:
g_n – is a characteristic value for loading from self weight;
g_{tn} – characteristic value of weight of heat insulation on the roof;
p_v^n - characteristic value of under pressure (vacuum) under the dome;
s_n – characteristic value of snow loading on the roof;
p_o^n - characteristic value of over pressure under dome;
γ_{fg} , γ'_{fg} , γ_{ft}, γ_{fv}, γ_{fs}, γ_{fa}, γ_{fw} – coefficients of overloading;
ψ_{c} – coefficient for combination of 2 or 3 temporary loads with short duration [2];
w_n – characteristic value of wind load on the dome.

3. Forces in the dome’s elements

Spatial steel domes with radial girders and circular elements are many times statistically undetermined spatial systems, which forces into elements difficultly can be calculated through manual solution. The precise calculation of the elements of steel structure of dome is done with use of suitable software, considering the stiffness of the elements and flexibility of the joints.

3.1. Forces in the elements of dome, manual solution

The preliminary calculations of the section of elements of spherical domes with radial girders and circular elements can be done on the base of pin - joint system, according to [1].

The biggest axial force S_i (pressure in q_1 and tension in q_2) in the dome’s girders is done when a full design load operates on its whole surface. The efforts in the girder immediately upon the ***i***-th joint are calculated according to the formula:

$$S_i = \frac{A_i.q}{n.\sin\alpha_{\mathrm{i}}} , \tag{3.1}$$

where:
A_i - is a circle surface, limited by the ***i***-th ring;
q – loading of combination q_1 or q_2;
n – number of main radial girders in the dome;
α_{i} – angle between the tangent to the girder in the ***i***-th connection in the horizontal plane.

Except an axial forces in the girders there is and a bending moment caused by distributed loads in the fields, limited by the steel structure. (Fig. 2).

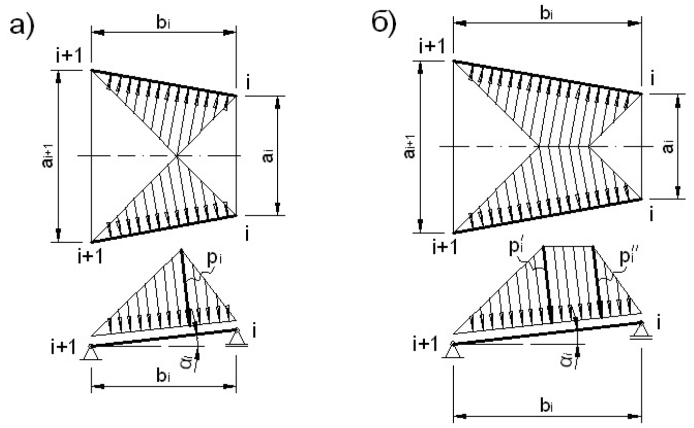

Fig. 2 Scheme for girders loading
a) with triangle shaped load б) with trapezium shaped load

When $\bar{a}_i \geq b_i$, on the girder accepted as a simply supported beam with opening $\boldsymbol{b_i}$, operates a load distributed according to the triangle low. (Fig. 2 – a). The maximum bending moment $\boldsymbol{M_{max}}$ would be calculated as in the simply supported beam, with the expression:

(3.2)
$$M_{\max} = \frac{1}{12} p_i . b_i^2$$

When $\bar{a}_i < b_i$, the distributed load has a trapezium shaped outline (Fig. 2 – б) and the value of the of maximum bending moment $\boldsymbol{M_{max}}$ is:

(3.3)
$$M_{\max} = \frac{1}{24} p_i . \left(3.b_i^2 - \bar{a}_i^2\right).$$

The value of $\boldsymbol{p_i}$ in the formulas (3.2) and (3.3) is calculated by summing up of the loadings of two neighbouring fields according to the schemes shown on Fig. 2

In the vertical plane main roof girders are measured according to compressed with a bending moment (when the load is $\boldsymbol{q_1}$) and/or tensioned with a bending moment (when the load is $\boldsymbol{q_2}$).

When there is a steel sheets welded to the construction the stability of the girders in horizontal plane is assured.

Where there is not a welded to the construction steel sheets and roof inclination is i > 1:16, the girders must be checked for general loss of stability between stiffening points.

The maximum of compression forces $\boldsymbol{T_i}$ in the $\boldsymbol{i}$-th joint, according to the described in [1] methodology appeared when there is a snow loading out of the circle of exanimated ring (Fig. 3). Its calculation is according to the formula:

(3.4)
$$T_i = \frac{1}{2.n.\sin\psi} . \left(\mathrm{cotg}(\alpha_i).q_1'.A_i - \mathrm{cotg}(\alpha_{i+1}).\left(q_1.(A_{i+1} - A_i) + q_1'.A_i\right)\right),$$

where:

(3.5)
$$q_1' = \gamma_{\mathrm{fg}}.g_n + \gamma_{\mathrm{ft}}.g_{tn} + \gamma_{\mathrm{fv}}.p_v^n \downarrow$$

(3.6)
$$2.\psi = \frac{2.\pi}{n}, \text{ [rad]}$$

in which **n** is the number of the main radial girders (Fig. 1).

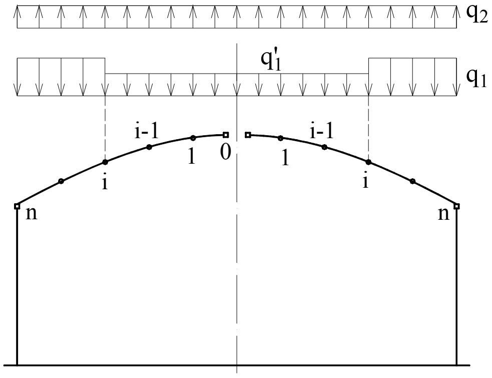

Fig. 3 Calculation of the forces in the intermediate rings.

(3.7) $$T_i' = \frac{q_2}{2.n.\sin\psi}.\left(\mathrm{cotg}(\alpha_{i+1}).A_{i+1} - \mathrm{cotg}(\alpha_i).A_i\right)$$

Bending moments appear in ring’s elements as a result of roof loading. For its determination the ring’s elements are accepted as simply supported beams with supporting distance equal to distance between joints.

Elements of the rings are measured to be able to bear compression forces with a bending moments or tension forces with a bending moments, depending on loading on them (q_1 or q_2). It is accepted that their effective length is equal to the geometrical. Maximum bending moments are calculated on formulas, analogical on (3.2) и (3.3) .

3.2 Forces in the dome elements in the numerical solution

When a numerical solution (FEA) is used for spatial domes with radial girders and circular elements, a spatial analytical model can be created, using convenient software. The radial girders and ring elements are inserted in this model as frame elements with their real geometric characteristics and materials. With not a serious error the radial girders outline the curved line through a field. Between its joints the elements are straight. The ring’s elements are turned in relation to their axis in the way that their upper flanges touch to the surface of the dome.

The transfer of the loading on the structure is simulated by loading areas, according to Fig. 2. The loads from over pressure, vacuum, and wind suction are perpendicular to the main axis of the frame elements. Loading caused by its self weight, heat insulation and snow is directed to the earth gravity.

In order to reporting the possibility of deformation of the top angle in horizontal direction, additionally, except steel structure of roof domes, it is created a model of top angle and a part of cylindrical steel shell.

4. Analytical and numerical research of spatial steel domes

Several steel domes with circular base are examined. They have different heights of over passing and diameters **D** of the base. They are situated in different part of the Earth Globe and the loads that impact them are not the same. Steel plates are not welded to the roof’s structure so we will research only on the load combination **q_1**.

The examined in the current research domes are designed in the real life. A part of them are constructed and the rest of them are under construction now.

Firstly the examined domes are calculated analytically through the described in point 3.1 methodology. Based on the calculated forces **S_i** , **T_i** and **M_{max}** and using the methodology in EC - 3 steel hot rolled sections are calculated which can bear them. After that with a appropriate software was created a relative spatial model in which are used analytically calculated steel sections.

5. Comparison of the obtained results

In the tables below it is shown a part of examined in the current research domes their diameters ***D*** in the base. Axial efforts and bending moments calculated with analytical and FEA solutions are compared.

Table 1 - Tank T017, Martinique, France

Volume *V* = 810 m^3 Diameter *D* = 12 m Height *H* = 7,2 m

Radial girders - UPN100

Field	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**M,** kN.m	**ΔN**	**ΔM**
1	-14,503	0,4026	-20,231	0,967	39,50	140,19
2	-29,007	1,0576	-28,02	1,157	3,52	9,40
3	-38,172	0,435	-39,753	2,683	4,14	516,78
4	-44,925	0,1845	-35,131	2,474	27,88	1240,92

Circular elements - UPN50-65

Circular elements	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**M,** kN.m	**ΔN**	**ΔM**
1	-44,52	0,115	-19,44	0,109	129,01	5,50
2	-29,57	0,717	-31,41	0,342	6,22	109,65
3	-21,4	1,23	15,23	0,745	40,51	65,10

Table 2 - Tank T105, Сливен, Bulgaria

Volume *V* = 5000 m^3 Diameter *D* = 22,8 m Height *H* = 11,95 m

Radial girders – IPN 140

Field	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**M,** kN.m	**ΔN**	**ΔM**
1	-25,859	0,510	-38,86	2,685	50,28	426,47
2	-46,587	1,107	-42,698	2,260	9,11	104,16
3	-61,898	2,414	-59,96	2,780	3,23	15,16
4	-70,683	1,482	-79,419	4,170	12,36	181,38
5	-85,059	0,705	-90,721	6,401	6,66	807,69
6	-94,475	0,274	-85,415	6,086	10,61	2119,55

Circular elements - UPN50-65

Circular elements	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**N,** kN	**ΔN**	**ΔM**
1	-16,968	0,067	-178,460	0,091	951,74	35,82
2	-100,058	0,316	-105,960	0,445	5,90	40,82
3	-101,097	0,546	-103,070	0,907	1,95	66,12
4	-68,273	0,681	-86,370	1,398	26,51	105,29
5	38,096	0,729	-70,750	2,039	85,72	179,70

Table 3 - Tank T016, La Reunion, France

Volume V = 14650 m3 Diameter D = 36 m Height H = 14,4 m

Radial girders - IPN200

Field	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**M,** kN.m	**ΔN**	**ΔM**
1	-36,794	1,9583	-48,14	3,897	30,84	99,00
2	-65,583	3,8738	-64,02	8,13	2,44	109,87
3	-82,019	2,2639	-82,595	6,335	0,70	179,83
4	-96,989	1,6051	-103,76	4,585	6,98	185,65
5	-109,91	1,104	-122,063	4,869	11,06	341,03
6	-121,49	0,815	-135,68	6,411	11,68	686,63
7	-132,05	0,633	-142,45	6,668	7,88	953,40
8	-141,82	0,496	-142,215	5,865	0,28	1082,46

Circular elements - IPN100-120

Circular elements	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**M,** kN.m	**ΔN**	**ΔM**
1	-257,23	0,056	-126,69	0,296	103,04	428,57
2	-186,55	0,288	-157,16	0,363	18,70	26,04
3	-152,09	0,619	-171,34	0,528	12,66	17,23
4	-130,13	0,974	-144,61	0,694	11,13	40,35
5	-114,94	1,311	-107,78	0,831	6,64	57,76
6	-102,91	1,6793	-62,05	0,904	65,85	85,76
7	-93,3	1,9345	19,86	1	369,79	93,45

Table 4 - Tank T01, Gent, Belgium

Volume V = 40000 m^3 Diameter D = 40,0 m Height H = 32 m

Radial girders - IPE200

Field	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**M,** kN.m	**ΔN**	**ΔM**
1	-38,096	3,23	-62,726	2,625	64,65	23,05
2	-70,31	4,036	-80,308	8,281	14,22	105,18
3	-91,756	2,252	-96,243	8,598	4,89	281,79
4	-109,039	1,625	-113,023	7,906	3,65	386,52
5	-123,969	1,237	-127,712	7,672	3,02	520,21
6	-137,271	0,89	-143,772	8,904	4,74	900,45
7	-149,397	0,685	-154,986	11,032	3,74	1510,51
8	-160,617	0,553	-161,214	11,566	0,37	1991,50
9	-171,114	0,461	-161,965	11,059	5,65	2298,92
10	-180,977	0,378	-156,914	7,11	15,34	1780,95

Circular elements - UPN140

Circular elements	manual solution		FEA with SAP2000		Difference, %	
	N, kN	**M,** kN.m	**N,** kN	**M,** kN.m	**ΔN**	**ΔM**
1	-373,32	0,029	-144,59	0,541	158,19	1765,52
2	-269,93	0,181	-157,14	0,278	71,78	53,59
3	-221,32	0,404	-151,58	0,299	46,01	35,12

4	-191,54	0,68	-139,7	0,359	37,11	89,42
5	-169,91	0,897	-147,19	0,421	15,44	113,06
6	-153,71	1,239	-111,93	0,566	37,33	118,90
7	-140,86	1,47	-68,52	0,694	105,58	111,82
8	-130,3	1,659	-18,49	0,79	604,71	109,95
9	-120,95	1,821	-59,18	0,859	104,38	111,98

6. Conclusions

6.1 Axial efforts in the radial girders increase from the dome’s centre toward its periphery (base) in the analytical as well as in the FEA solution;

6.2 The maximum values of axial forces calculated with the analytical research are bigger than those one calculated numerically. The differences do not exceeds 65%;

6.3 Using the analytical solution it is calculated that the biggest bending moments in the radial girders are near to the dome centre. More far from the centre where the rings are located closely the values of bending moment decrease. With the FEA solution the biggest bending moments in the radial girders are close to the periphery (base) of the dome.

6.4 Bigger values for bending moments in the radial girders are obtained with numerical solution. The differences of the value often can overpass 1600 %;

6.5 Using the analytical solution the biggest compressive forces in the ring elements are in the first ring counting from the dome centre. With the loading combination q_1 all axial forces in the rings are compressive;

6.6 Using the numerical solution the biggest calculated axial compressive forces in the ring elements usually are in the middle between centre and base of the dome. Often when the loading combination is q_1, the axial forces in the ring closest to the periphery are tensional;

6.7 The difference in the values of bending moments in the ring elements calculated using the numerical and analytical methods could exceeds 100%;

6.8 The sections of the steel construction of the dome calculated through described above analytical methodology are close to the really necessary. Often the sections calculated by this way are more safe but it is not recommendable to calculate them only analytically. FEA solutions give the possibility to improve the steel structure and to “detect” different asymmetric load on the domes.

LITERATURE

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