Analysis I Test №1, var2
Date : _ _ _ _ _ _

Student (names):_ **, faculty №** _ _ _ _ _

Solve the following problems :

Problem 1. Calculate:

a) $\frac{(2-3i)^3}{(4+i)^3}$ (4p.); b) $\sqrt[5]{(1-\sqrt{3}i)^{10}}$ (6p.).

Problem 2.

a) Find the quotient $q(x)$ and the remainder $r(x)$ from the division of polynomials $p(x)$ and $s(x)$ ($p(x)=s(x)q(x)+r(x)$, $q(x)=?$, $r(x)=?$) if $p(x)=x^5-x^2+x-12$ and $s(x)=3x^2-2x+5$ and check the result ;(5p).

b) Solve the inequality $\frac{x^4-16x}{x^3-11x^2+38x-40} \geq 0$ (5p.).

Problem 3.

a) Find the limit: $\lim\limits_{n\to\infty}\left(\sqrt{n^2+2n-23}-3n\right)$ (4p.);

b) Using the two policemen theorem, find $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\ldots+\frac{1}{\sqrt{n^2+n}}\right)$

(6p.)

Problem 4. Find the limits

a $\lim\limits_{x\to\infty}\frac{\sin\left(\frac{5}{x}\right)}{\sqrt{4+\frac{3}{x}}-2}$ (5p.); b) $\lim\limits_{x\to 0}\frac{\ln\left(1-4x^2\right)}{\sqrt{4+5x^2}-2}$ (5p.).

For any successfully solved problem you get 10 points.
The corresponding marks are as follows: If total number of your points Σ is:
$\Sigma<10 \Leftrightarrow$ **Poor (2), for** $\Sigma=10 \Leftrightarrow$ **Satisfactory (3); for** $1\Sigma=20 \Leftrightarrow$ **Good (4), for** $\Sigma=30 \Leftrightarrow$ **Very good (5) and** $\Sigma=40 \Leftrightarrow$ **Excellent (6).**

$\Sigma =$ _ _ _ _ **Mark:**_ _ _ _ _ _ _ _ _ **Signature:** _ _ _ _ _ _

/ Ivan Dimitrov /

Analysis I Test №2, var1 2015,
Date : _ _ _ _ _ _

Student (names):_ **, faculty №** _ _ _ _ _
Solve the following problems :

Problem 1. Find the derivatives:

a) $y = x^3 e^x + \dfrac{\operatorname{arctg} x}{1+x^2} - 5\log_3 x$ (5p.); b) $y = x\ln^2(x + 3\cos 5x)$, $y'(0) = ?$ (5p.).

Problem 2. Find:

a) The equation of the tangent line to the curve at the given point: $y = -x^3 + 3x$ at $x_0 = 2$ (4p.);

b) The angle between given curves at the points of intersection
$y = x^3 - 2x$ and $y = x^2$ (6p.).

Problem 3. Using the L'Hospital's rule find the limits:

a) $\lim\limits_{x\to 0} \dfrac{e^{2x} - 1 - 2x - 2x^2}{x - \dfrac{\sin 3x}{3}} = ?$ (4p.);

b) $\lim\limits_{x\to\infty} \left(\dfrac{x+4}{x+1}\right)^{4x}$ (6p.).

Problem 4.

a) For the function $y = \dfrac{x^2 - 1}{x}$ find the intervals of monotony, local extrema and asymptotes (6p.);

b) Let $f(x) = x - 3\sqrt[3]{x^2} + 5$. Find the least and the greatest values of f when $x \in [-2, 5]$. (4p.)

For any successfully solved problem you get 10 points.
The corresponding ratings are as follows: If total number of your points Σ is:
$\Sigma < 10 \Leftrightarrow$ **Poor (2), for** $\Sigma = 10 \Leftrightarrow$ **Satisfactory (3); for** $\Sigma \le 20 \Leftrightarrow$ **Good (4), for** $\Sigma = 30 \Leftrightarrow$ **Very good (5) and** $\Sigma = 40 \Leftrightarrow$ **Excellent (6).**

$\Sigma =$ _ _ _ _ **Rating:**_ _ _ _ _ _ _ _ _ **Signature:** _ _ _ _ _ _

/ **Ivan Dimitrov** /

Analysis I Test №2 var2
Date : _ _ _ _ _ _

Student (names):_ **, faculty №** _ _ _ _ _
Solve the following problems :

Problem 1. Find the derivatives:

a) $y = x^3 \cos x + \frac{2^x}{2 - x^2} - 4\arccos x$ (5p.); b) $y = \frac{\sqrt{(3x^2 + 2x - 1)^3}}{e^{-x}}$, $y'(0) = ?$ (5p.).

Problem 2. Find:

a) The equation of the tangent line to the curve at the given point: $y = \ln(x^2 + 1)$ at $x_0 = 1$ (4p.);

b) The angle between given curves at the points of intersection

$y = \frac{16}{x}$ and $y = x^3$ (6p.).

Problem 3. Using the L'Hospital's rule find the limits:

a) $\lim\limits_{x \to 0} \frac{\sin 3x - \arcsin 3x}{x^3} = ?$ (4p.);

b) $\lim\limits_{x \to \infty} \left(x + 3\right)^{\frac{2}{x}}$ (6p.).

Problem 4.

a) For the function $y = \frac{2x^2}{x + 1}$ find the intervals of monotonicity, local extremums and asymptotes and make a sketch of the graph. (6p.);

b) Let $f(x) = \sqrt[3]{x^5} - 5x + 2$. Find the least and the greatest values of f when $x \in [-2, 3]$. (4p.)

For any successfully solved problem you get 10 points.
The corresponding ratings are as follows: If total number of your points Σ is:
$\Sigma < 10 \Leftrightarrow$ **Poor (2), for** $\Sigma = 10 \Leftrightarrow$ **Satisfactory (3); for** $\Sigma = 20 \Leftrightarrow$ **Good (4), for** $\Sigma = 30 \Leftrightarrow$ **Very good (5) and** $\Sigma = 40 \Leftrightarrow$ **Excellent (6).**

$\Sigma =$ _ _ _ _ **Rating:**_ _ _ _ _ _ _ _ _ **Signature:** _ _ _ _ _ _

/ Ivan Dimitrov /

Analysis I Test №3, var2, 2016
Date : _ _ _ _ _ _

Student (names):_ , faculty № _ _ _ _ _

Solve the following problems :

Problem 1. Find the integrals:

a) $\int \frac{x}{(1+3x^2)^4} dx$ (3p.); b) $\int x\sin(3x)dx$ (3p.).

Problem 2. Using suitable substitution, find the integrals:

$\int \frac{e^{2x}}{e^{2x}-5e^x+4} dx$ (6p.).

Problem 3. Calculate the following definite integral;

$\int_{-1}^{3} x\arctan(x)dx$ (6p.).

Problem 4.

a) Find the area bounded by the curves $x = y^2$, $x + y = 3$ (3p.);

b) Find the length of the arc from the curve γ given by the equations

$\gamma : x(t) = 5\cos t,\ y(t) = 5\sin t,\ z = 4t,\ t \in [0,\ 2\pi]$ (3p.)

Problem 5. Let $f(x)$ be a continuous and an odd function, defined on the interval $[-a,\ a]$, $a > 0$.

a) Prove that $\int_{-a}^{a} f(x)dx = 0$; b) find the integral $\int_{-3}^{3} (1001x + 5 + \arctan^7(x))dx$.

For any successfully solved problem you get 6 points.
The corresponding ratings are as follows: If total number of your points Σ is:
$\Sigma < 10 \Leftrightarrow$ **Poor (2), for** $10 \le \Sigma \le 16 \Leftrightarrow$ **Satisfactory (3); for** $17 \le \Sigma \le 21 \Leftrightarrow$ **Good (4),**
for $22 \le \Sigma \le 26 \Leftrightarrow$ **Very good (5) and** $27 \le \Sigma \le 30 \Leftrightarrow$ **Excellent (6).**

$\Sigma =$ _ _ _ _ **Rating:_ _ _ _ _ _ _ _ _ Signature: _ _ _ _ _ _**

/ Ivan Dimitrov /

Analysis I Test №2
Date : _ _ _ _ _ _

Student (names):_ **, faculty №** _ _ _ _ _
Solve the following problems :

Problem 1. Find the derivatives:

a) $y = \sqrt[3]{x}\cos x + \dfrac{3^x}{2+x^3} - 4\arcsin x$ (5p.); b) $y = \dfrac{\sqrt{(3x^2+2x+3)^5}}{e^{2x}}$, $y'(0) = ?$ (5p.).

Problem 2. Find:

a) The equation of the tangent line to the curve at the given point: $y = x^4 - 5x^2$ at $x_0 = 1$ (4p.);

b) The angle between given curves at the points of intersection

$y = \dfrac{1}{x}$ and $y = x^2$ (6p.).

Problem 3. Using the L'Hospital's rule find the limits:

a) $\lim\limits_{x\to 0} \dfrac{\sin 3x - \operatorname{tg} 3x}{x - x\cos 2x} = ?$ (4p.);

b) $\lim\limits_{x\to\infty} \left(x + e^x\right)^{\frac{1}{x}}$ (6p.).

Problem 4.

a) For the function $y = \dfrac{x^2+2x}{x-1}$ find the intervals of monotony, local extrema and asymptotes (6p.);

b) Let $f(x) = \sqrt[3]{x^2} + 4x + 2.$ Find the least and the greatest values of f when $x \in [-3, 2]$. (4p.)

For any successfully solved problem you get 10 points.
The corresponding ratings are as follows: If total number of your points Σ **is:**
$\Sigma < 10 \Leftrightarrow$ **Poor (2), for** $\Sigma = 10 \Leftrightarrow$ **Satisfactory (3); for** $\Sigma = 20 \Leftrightarrow$ **Good (4), for** $\Sigma = 30 \Leftrightarrow$ **Very good (5) and** $\Sigma = 40 \Leftrightarrow$ **Excellent (6).**

$\Sigma =$ _ _ _ _ **Rating:**_ _ _ _ _ _ _ _ _ **Signature:** _ _ _ _ _ _

/ Ivan Dimitrov /

Analysis I Test №3, var2, 2016
Date : _ _ _ _ _ _

Student (names):_ , faculty № _ _ _ _ _

Solve the following problems :

Problem 1. Find the integrals:

a) $\int \frac{x}{(1+3x^2)^4} dx$ (3p.); b) $\int x\sin(3x)dx$ (3p.).

Problem 2. Using suitable substitution, find the integrals:

$\int \frac{e^{2x}}{e^{2x}-5e^x+4} dx$ (6p.).

Problem 3. Calculate the following definite integral;

$\int_{-1}^{3} x\arctan(x)dx$ (6p.).

Problem 4.

a) Find the area bounded by the curves $x = y^2$, $x + y = 3$ (3p.);

b) Find the length of the arc from the curve γ given by the equations

$\gamma : x(t) = 5\cos t,\ y(t) = 5\sin t,\ z = 4t,\ t \in [0,\ 2\pi]$ (3p.)

Problem 5. Let $f(x)$ be a continuous and an odd function, defined on the interval $[-a,\ a]$, $a > 0$.

a) Prove that $\int_{-a}^{a} f(x)dx = 0$; b) find the integral $\int_{-3}^{3} (1001x + 5 + \arctan^7(x))dx$.

For any successfully solved problem you get 6 points.
The corresponding ratings are as follows: If total number of your points Σ is:
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for $22 \le \Sigma \le 26 \Leftrightarrow$ **Very good (5) and** $27 \le \Sigma \le 30 \Leftrightarrow$ **Excellent (6).**

$\Sigma =$ _ _ _ _ **Rating:_ _ _ _ _ _ _ _ _ Signature: _ _ _ _ _ _**

/ Ivan Dimitrov /

Analysis I Test №3, 2017
Date : _ _ _ _ _ _

Student (names):_ , faculty № _ _ _ _ _

Solve the following problems :

Problem 1. Find the integrals:

a) $\int x\sqrt[3]{(3x^2-2)}dx$ (3p.); b) $\int x\ln(x+2)dx$ (3p.).

Problem 2. Using suitable substitution, find the integrals:

a): $\int \frac{1}{\sqrt{1-2x}-\sqrt[3]{1-2x}}dx$ (6p.);.

Problem 3. Calculate the following definite integrals: ;

$\int_{-1}^{3}(2x-3)e^{-\frac{1}{3}x}dx$ (3p.).

Problem 4.

a) Find the area bounded by the curve $c_1: y=x^2-x,\ y=4x,\ x\geq 0$ (3p.);

b) Find the length of the arc from the curve γ given by the equations

$\gamma: x(t)=\frac{1}{3}\cos^3 2t,\ \ y(t)=\frac{1}{3}\sin^3 2t,\ t\in[0,\ \pi/4]$ (3p.)

Problem 5. Prove that

a) if $f(x)$ is an even function, defined on the interval $[-a,a]$ and continuous on this interval, then

$\int_{-a}^{a} f(x)dx=2\int_{0}^{a} f(x)dx$;(3p) b) find the integral $\int_{-\pi}^{\pi} x^2\cos x\,dx$ (3p)

For any successfully solved problem you get 6 points.
The corresponding ratings are as follows: If total number of your points Σ is:
$\Sigma<10\Leftrightarrow$ **Poor (2), for** $10\leq\Sigma\leq16\Leftrightarrow$ **Satisfactory (3); for** $17\leq\Sigma\leq21\Leftrightarrow$ **Good (4),**
for $22\leq\Sigma\leq26\Leftrightarrow$ **Very good (5) and** $27\leq\Sigma\leq30\Leftrightarrow$ **Excellent (6).**

$\Sigma=$ _ _ _ _ **Rating:_ _ _ _ _ _ _ _ _ Signature: _ _ _ _ _ _**

/ Ivan Dimitrov /