1. Given the matrix $A=\begin{pmatrix}-1 & 0 & 1\\ 0 & 1 & 0\\ -3 & -1 & 2\end{pmatrix}$. Find

a) $B=A^2-3A+2E$; 3p.
b) find the matrix A^{-1} :4p.
c) solve the matrix equation $AX=B$ where $B=\begin{pmatrix}-2 & 1\\ 2 & 0\\ 0 & -3\end{pmatrix}$ and check the result! 3p.

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2. Find the rank of the matrix C **where**

$$C=\begin{pmatrix}2 & 2 & 0 & 4 & 3\\ -2 & 3 & -1 & 5 & 1\\ -1 & 2 & 1 & 6 & 5\\ 3 & 1 & 2 & 5 & 7\end{pmatrix}$$. 10 p.

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3. Using the method of Gauss solve the following system of linear equations

$$\begin{cases} 2x_1 - x_2 + x_3 = 6\\ -2x_1+3x_2+x_3=-2\\ x_1-4x_2-x_3=0\end{cases}$$. Check the result! 10p.

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4. a) Find the determinant

$$\Delta(x)=\begin{vmatrix}x & -x & 1 & 2\\ 2 & x & -x & 1\\ 1 & 2 & x & -x\\ -x & 1 & 2 & x\end{vmatrix}$$;7p.

b) Solve the equation $\Delta(x)=0.$3p.

1. Given the matrix $A=\begin{pmatrix}1 & -2\\ 3 & -5\end{pmatrix}$. Find

a) $B=A^3-A+E$;3p.
b) find the matrix A^{-1} 3p.
c) solve the matrix equation $AXA^{-1}=C$ where $C=\begin{pmatrix}3 & 0\\ 2 & -3\end{pmatrix}$ and check the result! 4p.

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2. **Find the rank of the matrix** C **where**

$$C=\begin{pmatrix}2 & 3 & -2 & 3\\ 7 & 4 & 5 & -3\\ -5 & 5 & 3 & 2\\ 6 & 2 & 0 & 8\\ 3 & 1 & -2 & -7\end{pmatrix}$$. 10p.

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3. Using the method of Gauss solve the following system of linear equations

$$\begin{cases} 3x_1 - x_2 + x_3 = 2\\ -2x_1+3x_2+x_3=-4\\ x_1+3x_2+7x_3=6\end{cases}$$,. Check the result! 10p.

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3. a) Find the determinant

$$\Delta(x)=\begin{vmatrix}x & 1 & 2 & 3\\ 1 & x & 2 & 3\\ 1 & 2 & x & 3\\ 3 & 1 & 2 & x\end{vmatrix}$$; 7 p.

b) solve the equation $\Delta(x)=0.$ 3p.

LAAG **Test №2,var1**

Date : _ _ _ _ _ _

Student (names):_ **, faculty №** _ _ _ _ _

Solve the following problems:

Problem 1. Given the points $A(q,1,5), B(1,5,9), C(5,-1,2)$ and $D(3,1,0)$. :

a) For $q=1$ find the volume of tetrahedron $ABCD$ and the length of the height from the point B (5p.);

b) Find the value of parameter q in case that the vector $\overrightarrow{AD} \perp \overrightarrow{BC}$. (5p.).

Problem 2. Given the vectors: $\vec{a}=(1,2,3), \vec{b}=(1,3,4), \vec{c}=(-1,3,4), \vec{d}=(-1,-4,-5)$ and the point $M(1,2,3)$.

a) Check that the vectors $\vec{a}, \vec{b}, \vec{c}$ form a base in $\mathbb{R}^3$ (that means the three dimensional space) and find the coordinates of a vector $\vec{d}$ with respect this base (6p.);

b) Find the vertexes of trangular prism with one vertex at the point M and built on the vectors $\vec{a}, \vec{b}, \vec{c}$. (4p.)

Problem 3. Using the points from the Problem 1, find :

a) a vector which is colynear with the height from the vertex C of tetrahedron (4p.);

b) the value of parameter q if it is known that $\measuredangle\left(\overrightarrow{BD}, \overrightarrow{AC}\right)=\frac{\pi}{6}$ (6p.).

Problem 4. Given the matrix $A=\begin{pmatrix} 1 & -3 & -1 \\ -3 & 1 & 1 \\ -1 & 1 & 5 \end{pmatrix}$ and the vector $\vec{x}=\begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$.Find:

a) the vector $\vec{y}=A\vec{x}$ (3p.);

b) the eigenvalues and eigenvectors of the given matrix and check the result .(7p.)

For any successfully solved problem you get 10 points.
The corresponding ratings are as follows: If total number of your points Σ is:
$\Sigma<10 \Leftrightarrow$ **Poor (2), for** $\Sigma=10 \Leftrightarrow$ **Satisfactory (3); for** $\Sigma=20 \Leftrightarrow$ **Good (4), for** $\Sigma=30 \Leftrightarrow$ **Very good (5) and** $\Sigma=40 \Leftrightarrow$ **Excellent (6).**

$\Sigma=$ _ _ _ _ **Rating:**_ _ _ _ _ _ _ _ _ _ **Signature:** _ _ _ _ _ _

/ Ivan Dimitrov /

LAAG **Test №2, var. 2**
Date : _ _ _ _ _ _

Student (names):_ **, faculty №** _ _ _ _ _
Solve the following problems:

Problem 1. Given the points $A(1,3,5), B(-1,0,-7), C(\lambda,1,-3)$ and $D(2,1,5)$.

a) For $\lambda=-2$ find the volume of tetrahedron $ABCD$ and the length of the height from the point D (5p.);

b) Find the value of parameter λ in case that the vector $\sphericalangle\left(\overrightarrow{AD},\overrightarrow{BC}\right)=\frac{\pi}{4}$ (5p.).

Problem 2. Given the vectors: $\vec{a}=(\mu,2,3),\vec{b}=(5,3,-2),\ \vec{c}=(-9,-4,7)$.

a) Check that for $\mu=1$ the vectors $\vec{a},\ \vec{b},\ \vec{c}$ are linear dependent, also the vectors $\vec{a},\ \vec{b}$ are linear independent and find the representation of a vector $\vec{c}$ as a linear combination of vectors $\vec{a},\ \vec{b}$ (5p.);

b) If $\mu=-2$, find a vector $\vec{d}$ such that $\vec{d}.\vec{a}=4,\ \vec{d}\perp\vec{b}$ and $\vec{d}.\vec{c}=-1$. (5p.)

Problem 3. Using the points from the Problem 1, find:

a) $\cos\alpha$, where $\alpha=\sphericalangle\left(\overrightarrow{AB},\overrightarrow{AD}\right)$ (4p.);

b) the value of parameter λ if it is known that $\overrightarrow{AB},\overrightarrow{AC},\ \overrightarrow{AD}$ are coplanar . (6p.).

Problem 4. Given the matrix $A=\begin{pmatrix}-3 & 2\\ 2 & -3\end{pmatrix}$ and the vector $\vec{x}=\begin{pmatrix}-3\\ 1\end{pmatrix}$.Find:

a) the vector $\vec{y}=A\vec{x}$ (3p.);

b) the eigenvalues and eigenvectors of the given matrix and calculate $P(A)=A^2+6A+5E$, where $E=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$.(7p.)

For any successfully solved problem you get 10 points.
The corresponding ratings are as follows: If total number of your points Σ **is:**
$\Sigma<10\Leftrightarrow$ **Poor (2), for** $\Sigma=10\Leftrightarrow$ **Satisfactory (3); for** $\Sigma=20\Leftrightarrow$ **Good (4), for** $\Sigma=30\Leftrightarrow$ **Very good (5) and** $\Sigma=40\Leftrightarrow$ **Excellent (6).**

$\Sigma=$ _ _ _ _ **Rating:**_ _ _ _ _ _ _ _ _ _ **Signature:** _ _ _ _ _ _
/ Ivan Dimitrov /

LAAG Test №3, var. 2
Date : _ _ _ _ _ _

Student (names):_ **, faculty №** _ _ _ _ _
Solve the following problems:

Problem 1. Given the points $A(1,5), B(-1,0), C(1,-3)$.

a) Find the equations of the straight lines (AB) and (AC) and the of the height of the triangle ABC from the point C (5p.);

b) Find the equation of a line on which lies this height and the slope of this line . (5p.).

Problem 2. Given the vectors: $\vec{a}=(2,2,3), \vec{b}=(5,3,-2)$ and a point $M(-9,-4,7)$..

a) Find the equation of a plane α, $\alpha \parallel \vec{a}, \vec{b}$ and $M \in \alpha$. (4p.);

b) Find the equations of the straight line g, $g \perp \alpha$ and $M \in g$. Moreover find the intersecting point of g with the plane Oyz.. (6p.)

Problem 3. Using the points from the Problem 1, find:

a) Coordinates of a point D, such that the triangle ABD is rectangular with $\measuredangle D = 90^0$. (4p.);

b) Coordinates of a center and a radius of the outside circle for the triangle ABD. (6p.).

Problem 4. Given the curve : $25x^2 - 4y^2 = 100$. Find:

a) the type of the curve. (4p.);

b) semi-axes, focuses and asymptotes of the given curve.(6p.)

For any successfully solved problem you get 10 points.
The corresponding ratings are as follows: If total number of your points Σ is:
$\Sigma < 10 \Leftrightarrow$ **Poor (2), for** $\Sigma = 10 \Leftrightarrow$ **Satisfactory (3); for** $\Sigma = 20 \Leftrightarrow$ **Good (4), for** $\Sigma = 30 \Leftrightarrow$ **Very good (5) and** $\Sigma = 40 \Leftrightarrow$ **Excellent (6).**

$\Sigma =$ _ _ _ _ **Rating:**_ _ _ _ _ _ _ _ _ _ **Signature:** _ _ _ _ _ _

/ Ivan Dimitrov /