STEEL DOMES WITH RADIAL GIRDERS AND CIRCULAR ELEMENTS ON CIRCULAR BASE. HOW TO DESIGN THEM

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ABSTRACT

Domes on a circular base have a wide field of application – silos, tanks, warehouses for wastes, sport, and exhibition halls. They are light, beautiful and can bridge big spans, assuring a free space without internal columns. The steel domes have been applied successfully to spans with a diameter up to 50 m. They are easy for prefabrication, transportation and mounting when compact hot-rolled sections are used.

The considerable part of the domes is used to cover steel tanks for storage of petrol products. With the increasing of the capacities of the steel tanks, their diameter grows up, too. It leads from one side to bigger forces in structural elements and on the other side – to effects that could be neglected in the smaller tanks. As a result, more precise accounting of loads and the real behaviour of their supporting structure is needed. Unfortunately, in the leading world’s standards for steel tanks design API Std. 650, EN 14015:2004 and EN 1993-4-2:2007, there are no instructions how to design steel domes with radial girders and circular elements. In the present paper, based on his experience and research in the field, the author gives the main requirements which should be matched during the design of these spatial facilities.

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1. Design of the Steel Structure of the Roof

Traditionally, the structure of steel domes on a circular base is composed by radial girders, circular elements and roof-cover plates, see Fig. 1.

These elements should be stable, with enough bearing capacity during mounting and exploitation. Unfortunately, in the leading standards for steel tanks design API Std. 650 [1], EN 14015:2004 [7] and EN 1993-4-2:2007 [6], there are no instructions how to design steel domes with radial girders and circular elements.

1.1. Design for Mounting Condition

First, the elements of the roof structure should be designed for mounting condition. At the moment, the most used methods for mounting are:

a) by using a temporary central column – in this case one end of the main radial girders are placed on the upper edge of the shell and the other ends – on a temporary central column, see Fig. 2a;

b) by floating of the roof – steel tank is being filled slowly with a fresh water and the self-supporting dome floats on it as a raft, see Fig. 2b;

c) by assembling on the ground – roof is assembled on temporary supports and after that is lifted and mounted with a crane as one piece, see Fig. 2c.

Regardless of which of the above-mentioned mounting methods is chosen, there always exists a condition when the radial roof girders have only two supports. In this situation workers move on it and do mounting works. The design scheme is a simply supported beam with a span, equal to the distance between the top angle and the central ring of the roof, see Fig. 3. As a result of applied loads and supporting conditions during execution, forces which often exceed these during exploitation appear in the radial girders.

Ordinates of the distributed loads above the supports, see Fig. 3, could be calculated according to the formulae:

\[ P_1 = \left( \gamma_{G,\text{sup}} g + \gamma_Q q_{\text{ca,k}} \right) a_1 \]  
\[ P_2 = \left( \gamma_{G,\text{sup}} g + \gamma_Q q_{\text{ca,k}} \right) a_n \]
where $\gamma_{G,\text{sup}} = 1.35$ is the partial coefficient of loading by self-weight of roof structure and cover plates, when this load is unfavourable, according to EN 1990:2003 [2];

- $g$ – characteristic value of self-weight of the roof structure and cover plates;
- $\gamma_Q = 1.5$ – partial coefficient of loading by live loads on roof structure;
- $q_{ca,k} = 1.0 \text{kN/m}^2$ – characteristic value of the live loads due to workers and equipment on the roof during mounting, according to the standard EN 1991-1-6:2005 [4];

- $a_1$ – distance between the radial girders in the place of join with the central ring of the roof;

- $a_n$ – distance between the radial girders in the place of join with the shell, see Fig. 1b.

Figure 2. Mounting of self-supporting roofs

- a) using of a temporary central column
- b) when roof floats
- c) assembling on the ground and lifting as one piece
The practical experience of the author shows that on the roof there are not plenty of workers and equipment; neither during the mounting operations nor the exploitation. According to him, the value of live load $q_{ca,k}$ from workers on the roof should be multiplied by the reduction coefficient $\alpha_A$:

$$\alpha_A = \frac{A_0}{A} \leq 1,$$

where $A_0 = 10 \text{ m}^2$;

$A$ – the area from which live load $q_{ca,k}$ is transmitted to considered element of roof structure, in $\text{m}^2$.

In order to avoid very low values of the live load by workers, the inequality below should be fulfilled:

$$\alpha_A q_{ca,k} \geq 0.35 \text{ kN/m}^2.$$  

Maximum value of bending moment $M_{y,max}$ in the radial girder is obtained at distance $x$ from the right support. According to Venkov [9] the value could be determined by the formula:

$$M_{y,max} = \frac{xl}{6} \left( p_2 + 2p_1 \right) - \frac{x^2}{2} p_1 - \frac{x^3}{6l} \left( p_2 - p_1 \right),$$

where $l = r - r_1$ is the distance between supports of the radial girders during the mounting works, see Fig. 3;

$r$ – radius of the cylindrical shell of the tank;

$r_1$ – radius of the central ring of the roof;

$$x = \frac{l}{p_2 - p_1} \left[ \sqrt{p_1^2 + \left( p_2 - p_1 \right) \left( p_2^2 + 2p_1 \right)} - p_1 \right].$$

Figure 3. Static scheme and loading during the mounting
When the steel cover plates are welded to upper flanges of supporting structure during mounting works, the elements should be checked for strength only, as follows:

– by normal stresses, caused by bending moments

\[
\frac{M_{y,Ed}}{M_{y,Rd}} \leq 1,
\] (7)

where \( M_{y,Ed} \) is the design value of the bending moment by axis “y-y” in radial girder; 
\( M_{y,Rd} \) – design value of the resistance to bending by axis “y-y”, determined according to the standard EN 1993-1-1:2004 [5].

– by shear forces

Steel sections should be checked for shearing by formula:

\[
\frac{V_{Ed}}{V_{pl,Rd}} \leq 1,
\] (8)

where \( V_{Ed} \) is the design value of shear force in steel section; 
\( V_{pl,Rd} \) – design plastic shear resistance, see standard EN 1993-1-1:2004 [5].

When the roof cover plates are welded under the supporting structure, i.e. to tensioned flange of sections, or when such cover plates are missing during mounting, elements should be checked for strength by formulas (7) and (8), and for lateral torsional buckling by equation:

\[
\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1,
\] (9)

where \( M_{b,Rd} \) is the design value of buckling resistance, determined according to standard EN 1993-1-1:2004.

Design lengths \( l_{LT} \) of the elements of radial girders during the verifications for lateral torsional buckling in mounting condition are equal to the distance \( l_i \) between joints of the radial girders and circular elements, see Fig. 1.

1.2. Design for Condition of Exploitation

During the exploitation, finished domes with radial girders and circular elements are many times statically indeterminate spatial systems. The internal forces in them cannot be easily determined through manual calculations. Exact calculation of forces in the elements should be done with appropriate software, with accounting of stiffness of the elements and flexibility of their joints.

1.2.1. Manual (Analytical) Calculations

Preliminary determination of the sections of the elements in domes with radial girders and circular elements could be done on the base of the model of computation, proposed by Sokolov [12]. In this model the connections between all frame elements are hinge and load is symmetrically distributed toward the vertical axis.
The biggest axial force $S_i$ (compression at $q_1$ and tension at $q_2$) in joint $i$ of the radial girders of the domes is calculated when full design load is applied on the whole surface above the joint $i$. The force in girders should be calculated according to the formula:

$$S_i = \frac{q A_i}{n \sin \alpha_i},$$

(10)

where $A_i$ is the area of the circle, limited by the ring $i$;
$q$ – applied load by design load combination $q_1$ or $q_2$;
$n$ – the number of main radial girders in the dome.

$$q_1 = \max \left\{ \gamma_{G,\text{sup}} (g + g_t) + \gamma_Q s + \gamma_Q \psi_{0,1} p_v \downarrow, \gamma_{G,\text{sup}} (g + g_t) + \gamma_Q P_v + \gamma_Q \psi_{0,1} s \downarrow \right\},$$

(11)

$$q_2 = \max \left\{ \gamma_Q P_o + \gamma_Q \psi_{0,1} w_n - \gamma_{G,\text{inf}} (g + g_t) \uparrow, \gamma_Q w_n + \gamma_Q \psi_{0,1} P_o - \gamma_{G,\text{inf}} (g + g_t) \uparrow \right\},$$

(12)

where $g_t$ is the characteristic value of the weight of heat insulation on the roof (if any);
$p_v$ – characteristic value of negative internal pressure under the dome;
$s$ – characteristic value of snow load on the roof;
$p_o$ – characteristic value of internal pressure under the dome;
$\gamma_{G,\text{sup}} = 1.35$ is the partial coefficient of loading by permanent actions, when these loads are unfavourable, according to EN 1990:2003 [2];
$\gamma_{G,\text{inf}} = 1.0$ – partial coefficient of loading by permanent actions, when these loads are favourable;
$\psi_{0,1}$ – coefficient for combination of the variable loads, according to EN 1990:2003;
$w_n$ – average characteristic value of the wind load on the whole dome roof.

The loads on the roof cause bending moment in the radial girders. Having in mind accepted model of computation, i.e. every frame element is connected to the next with hinges; all elements have a length equal to the distance between the joints.

When the inequity $\tilde{a}_i \geq b_1$ is matched, on the radial girders, accepted as a simply supported girders with a span $b_i$, acts a triangular load, see Fig. 4a. Maximum bending moment $M_{y,\text{max}}$ to axis “y-y” of section is:

$$M_{y,\text{max}} = \frac{1}{12} p_i b_1^2.$$

(13)

When $\tilde{a}_i < b_1$, the distributed load is trapezoidal, see Fig. 4b, and the value of the maximum bending moment $M_{y,\text{max}}$ to the axis “y-y” is:

$$M_{y,\text{max}} = \frac{1}{24} p_i \left(3b_1^2 - \tilde{a}_i^2\right).$$

(14)

The value $p_i$ of the formulas (13) and (14) is obtained by summing the loads from two adjacent fields, as is shown on Fig. 4.
The main radial girders of dome roof should be designed for condition of exploitation for:

a) strength:
   – as members in bending and axial compression – when the loading combination is $q_1$;
   – as members in bending and axial tension – when the cover plates are welded to the structure and loading combination is $q_2$;
   – for shearing – this verification rarely is valid.

b) for general loss of stability – as members in bending and axial compression, by formulae:

$$\frac{N_{Ed}}{N_{Rk}} + k_{yy} \frac{M_{y,Ed}}{M_{y,Rk}} \leq 1; \quad (15)$$
\[
\frac{N_{\text{Ed}}}{N_{Rk}} + k_{zy} \frac{M_{y,\text{Ed}}}{M_{y,Rk}} \leq 1,
\]

(16)

where \( N_{\text{Ed}} \) is the design value of axial force in the radial girder;
\( N_{Rk} \) – characteristic resistance to normal force of the critical steel section;
\( \chi_y \) and \( \chi_z \) – reduction factors of flexural buckling related to the axes “y-y” and “z-z”, determined according to the standard EN 1993-1-1:2004 [5];
\( \gamma_{LT} \) – reduction factor for lateral – torsional buckling;
\( k_{yy}, k_{yz}, k_{zy}, k_{zz} \) – coefficient of interaction;
\( \gamma_{M1} \) – partial coefficient for resistance of a member’s loss of stability.

The values of the \( N_{Rk} \) and \( M_{y,Rk} \) should be calculated through the following expressions:

\[
N_{Rk} = f_y A;
\]

(17)

\[
M_{y,Rk} = f_y W_y,
\]

(18)
in which \( f_y \) is characteristic value of the yield strength of the steel;
\( A \) – the area of cross-section;
\( W_y \) – section modulus of steel section, related to axis “y-y”.

When steel sheets are continuously welded to the roof structure, it can be assumed that the stability of the radial girders in a horizontal plane is ensured.

When the steel sheets are not welded to the roof structure and the slope of the roof \( i > 1:6 \), the radial girders should be checked for the general loss of stability between the stabilizing points, i.e. design length of the elements of radial girders is \( l_y = l_z = l_i \), see Fig. 1.

Maximum compressive force \( T_{i,c} \) in ring \( i \) of the frame model with hinge joints of Sokolov [12] is calculated when snow load is outside the examined ring, see Fig. 5. The calculation should be done according to the formula:

\[
T_{i,c} = \frac{1}{2n \sin(\psi)} \left[ \cotg(\alpha_i) q_i A_i - \cotg(\alpha_{i+1}) \left( q_i (A_{i+1} - A_i) + q_i' A_i \right) \right],
\]

(19)

where:

\[
q_i' = \gamma_{G,\text{sup}} (g + g_t) + \gamma_{Q} p_v \downarrow.
\]

(20)

Maximum tension force \( T_{i,t} \) in the ring \( i \) of the roof is accounted when the whole surface of the roof is loaded by forces in combination \( q_2 \), see Fig. 5. The value of force \( T_{i,t} \) could be accounted according to the expression:

\[
T_{i,t} = \frac{q_2}{2n \sin(\psi)} \left( \cotg(\alpha_{i+1}) A_{i+1} - \cotg(\alpha_i) A_i \right).
\]

(21)
Figure 5. Determining the forces in the intermediate rings

Loads on the roof generate bending moments in circular elements. For their calculation, the circular elements are considered as simply supported beams with a span \(a_i\), equal to the distance between two adjacent main girders, see Fig. 1. The shape of load could be triangular or trapezoidal, depending on the dimensions of the fields. The maximum value of bending moments should be calculated according to the formulas, analological to (13) and (14).

The circular elements should be designed according to EN 1993-1-1:2005 [5] for strength and for loss of stability.

When the roof cover plates are continuously welded to the structure, it could be accepted that the stability of the circular elements toward the axis “z-z” (the “weak” axis) is ensured.

When the steel sheets are not welded to the structure and the roof slope is \(i > 1:6\), the circular elements should be checked for the general loss of stability between stabilizing points, i.e. design length is \(l_y = l_z = a_i\), see Fig. 1.

In a research from 2010 about reliability of the analytically (manually) calculated values of the forces \(S_i\), \(T_i\) and \(M_{y,max}\), Zdravkov [10] found the following:

a) in case of computational model of frame with hinge joints, the manual (analytical) calculations show higher values of axial forces in the elements;

b) bigger values of the bending moments in the radial girders are obtained in the numerical (FEA) solution;

c) bigger values for bending moments in the circular elements are calculated in manual (analytical) solution;

d) the sections of steel structure of the dome, calculated by the analytical methodology, described above, are close to the really needed. Often the sections calculated on this manner are more secure.

1.2.2. Numerical Calculations

During this type of analysis a spatial computational model should be created, using appropriate software. Radial girders and circular elements should be entered as frame elements.
On a first step, their sections are determined by frame model with hinge joints and symmetrically distributed load of Sokolov [12]. The circular elements should be rotated toward their longitudinal axis so that their upper flange tangles to the dome surface. When the cover plates are not welded to the structure, it is not recommended to enter them as a structural element in the model. In order to account flexibility of the supports of the dome in horizontal direction is good idea top angle and part of cylindrical steel to be modelled additionally, see Fig. 6.

The transition of the loading from the cover plates to the structure could be simulated using loading areas, according to Fig. 4. Loads by over pressure, vacuum and wind suction are perpendicular to longitudinal axes of the frame elements. Loading by self-weight, heat insulation and snow is in the direction of gravity.

Snow load on roof of the tank could be determined according to standard EN 1991-1-3:2003 [3]. For long- or short-term design condition it could be calculated by the formula:

\[ s = \mu_i C_e C_t s_k, \]

where \( \mu_i \) is the snow load shape coefficient, see Fig. 7;
- \( C_e \) – exposure coefficient;
- \( C_t \) – thermal coefficient;
- \( s_k \) – characteristic value of the snow on the ground.

Due to the uneven sunshine and/or transfer of snow masses from the wind, it is normal to have uneven loading from snow on the roof. Unfortunately, in the standards EN 1991-1-3 [3] and EN 14015:2004 [7] it is not shown a case for uneven snow loading on the roof with circular base in the plan. Here the scheme mentioned in the API Std 650 [1] can be used, see Fig. 7b:

- snow upon 135° roof zone, coefficient of snow loading \( \mu_i = \mu_3 = 1.5; \)
– on the rest of the roof surface from 225° there is no snow.

Figure 7. Distribution of the snow loading on the roof with a circular base

The forces in the elements due to load combination \( q_1 \) and \( q_2 \) are accounted by the spatial numerical model. With these forces a recheck of the sections of radial girders and circular elements should be done. If it is necessary, the sections could be modified, but they should not be smaller than the sections calculated for the case of mounting.

In case of cover plates continuously welded to a structure, a flexural buckling to “weak” axis “z-z” is ensured when the stresses in non-welded flange are tensile.

When the steel sheets are not welded and the slope of the roof \( i > 1:6 \), the elements should be checked for general loss stability between the stabilizing points. On a first step computational length of the elements equal to the geometrical could be accepted, see Fig. 1. It is more or less true but is unprovable. For that reason, as a second step, the stability of the roof and its elements should be checked by General method of EN 1993-1-1:2005.

1.2.3. General Method for Buckling of Members

When a building structure is spatial (3D), all elements are loaded and work together. They influence and support each other. In that situation it is very difficult to determine which is the supporting and which – the supported member. It is impossible to calculate directly their effective lengths. These lengths are used in classic equations of standards to check elements for loss of stability. For that reason it is reasonable to use the General method, described in European standard EN 1993-1-1:2005 [5]. For the purpose of the analysis, buckling option is activated in used software. It is possible to observe the reserve of resistance before one particular element or the whole structure loses stability. The solution is linear, but taking into account the deformations in the roof structure.

Permanent loads as the self-weight of the structural elements and/or heat insulation on the roof should be entered first, in geometrical non-linear case. After that, in deformed by the permanent loads structure, the loading by under pressure (vacuum) \( p_v \) and snow \( s \) should be entered. The reserve of overall resistance \( K \) should be accounted for these temporary loads. The snow and under pressure are entered in the model with their design values.

Overall resistance to out-of-plane buckling for any structural component, conforming to the scope of the General method, can be verified by ensuring that:
\[
\frac{\chi_{op} \alpha_{ult,k}}{\gamma_{M1}} \geq 1,0,
\]

where \(\alpha_{ult,k}\) is minimum load amplifier of the design loads to reach the characteristic resistance of the most critical cross section of the structural component, considering its in-plane behaviour without taking into account lateral or lateral – torsional buckling. It can be calculated through the expression:

\[
\alpha_{ult,k} = \frac{f_y}{\left( \frac{M_{y,Ed} + N_{Ed}}{W_{el,y}} + \frac{1}{A} \right)^\gamma},
\]

in which \(A\) is the area of the composed section;

\(W_{el,y}\) – minimum elastic resistant moment toward the axis “y-y”.

\(\chi_{op}\) – the reduction factor for the non-dimensional slenderness \(\bar{\lambda}_{op}\), to take account of lateral and lateral torsional buckling.

The global non-dimensional slenderness \(\bar{\lambda}_{op}\) for the structural component should be determined by:

\[
\bar{\lambda}_{op} = \frac{\alpha_{ult,k}}{\alpha_{cr,op}},
\]

where \(\alpha_{cr,op}\) is the minimum amplifier for the in-plane design loads to reach the elastic critical resistance of the structural component with regards to lateral or lateral torsional buckling without accounting for the in-plane flexural buckling.

When the roof is loaded with the above described sequence of load application, the parameter \(\alpha_{cr,op}\) could be calculated by the expression:

\[
\alpha_{cr,op} = \frac{g \gamma_{G,inf} + K \left( g' \gamma_{Q} + \psi_0 P_v \gamma_{Q} \right)}{g \gamma_{G,sup} + s \gamma_{Q} + \psi_0 P_v \gamma_{Q}},
\]

where \(K\) is a buckling coefficient, reporting how many times the design values of the temporary loads \(p_v\) and \(s\) have to be increased, before a particular element or the whole structure loses stability.

The reduction coefficient \(\chi_{op}\) can be calculated by one of the following ways:

a) the minimum value of:

\(\chi_z\) – for lateral buckling by axis “z-z”, according to 6.3.1 of standard EN 1993-1-1, see Fig. 8;

\(\chi_{LT}\) – for lateral – torsional buckling due to bending moments \(M_{y,Ed}\), each calculated for the global non-dimensional slenderness \(\bar{\lambda}_{op}\).
b) a value interpolated between the values \( \chi \) and \( \chi_{LT} \) as determined in a) by using the formula for \( \alpha_{ult,k} \) corresponding to the critical cross section.

Figure 8. Deformed shape of roof when it loses stability

When the roof cover plates are not welded to the roof structure, they should not be included in computational model. When the steel sheets are welded to structure, on the side of safety is to include only a part of them, Zdravkov [8]. They are included on each side of the longitudinal angular welds, joining the section and steel sheets. Accepted width depends on the class of the steel section.

By a research on domes with radial girders and circular elements, done in 2016, Zdravkov [11] concluded that the successful check of radial girders for mounting condition significantly increases the probability that the spherical dome would not lose overall stability during operation.

2. Bracing

When the roof cover plates are not connected to the supporting structure, standard EN 1993-4-2 [6] requires to place braces between elements of roof structure. Their number and type are as follows:

a) when the tank has diameter \( D > 15 \text{ m} \) – at least two bays of diagonal braces (two couples of girders connected with grid elements) have to be placed. The sets of braced bays should be spaced evenly around tank’s circumference;

b) for braced roofs with diameter \( D \) between 15 and 25 m, an additional circumferential ring should be provided;

c) for braced roofs with diameter \( D > 25 \text{ m} \), two additional circumferential rings should be provided.

The bracing should be designed for a stabilizing force equal to 1% from the sum of normal forces in the stabilised elements.
In his research from 2016, Zdravkov [11] reached the following conclusions:

a) rigid joints between radial girders and circular elements are a much more appropriate solution than hinge joint between the elements and stabilizing braces between them;

b) in some cases rigid joints between elements of the roof structure can completely eliminate the need of using additional stabilizing braces.

3. Conclusion

The steel domes with radial girders and circular elements on a circular base are widespread in the practice. They are many times statically indeterminate systems, which in the past necessitated some simplifications in calculations. Nowadays, having knowledge, computers and appropriate software, we can design these structures in more reliable manner. The most important element of this trinity, as always, is the knowledge. Based on the experience of older colleagues and his researches in the field, the author has developed a methodology that has not been misleading so far. On the one hand, it is simple, logical, friendly to the designer, and on the other hand it is quite reliable.

REFERENCES