Analyzing the displacements of a pile frame to determine its transverse load-bearing capacity

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ABSTRACT

This paper presents a numerical study of the deformation properties of a regular pile group foundation frame, which consists of 9 piles with square cross-sections (40/40 cm) and spacing of 1,60 m (axis-to-axis). The length of the piles is 11,0 m and the thickness of the pile cap is 150 cm. Four separate analyses are carried out. Initially the pile group is calculated according to the procedure and requirements of the current Bulgarian Codes. After that a solution with elastic spring supports is presented. Spring stiffness is assumed to increase linearly with depth and stiffness-distribution coefficients are introduced. A linear-elastic plane stress analysis is performed using SAP2000 – with frame elements for the piles and membrane elements for the soil body. Finally nonlinear 2D and 3D analyses are carried out with PLAXIS 2D/3D, taking into account the plastic properties of soils. The Mohr-Coulomb failure criterion is assumed for all layers. The results are summarized and general recommendations are given for the simplified solutions, based on the more detailed nonlinear procedures. An essential observation is that load distribution differs significantly in the final stage of the plastic analysis, where the system is near a state of critical equilibrium, which corresponds to an ultimate limit state.

Keywords: pile foundations, horizontal force, bearing capacity, limit states, finite element method, soil-structure interaction

1 INTRODUCTION

In the case of pile foundations, “bearing capacity” often has the meaning of satisfying displacement/deformation criteria or in other words the serviceability limit states.

In terms of bearing capacity for vertical actions, there are a number of analytical solutions as well as direct analogies with results from in-situ tests. In transverse (or horizontal) direction, however, there are no clear criteria regarding ground bearing capacity. Ultimate limit states are often governed by structural failure or excessive displacements greater than a certain “structural” limit [1]. Structural failure often precedes ground failure, hence making the serviceability (SLS) and structural (STR) limit states essential.

Calculating the lateral resistance (bearing capacity) of a pile is no trivial task, and often the procedures specified in building codes are inconclusive and insufficient for soil-structure interaction analyses. This paper presents a comparative study of analysis methods for determining the load-displacement relation for a pile foundation frame, subjected to significant horizontal force.

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2 GEOMETRICAL AND GEOTECHNICAL PARAMETERS

The foundation frame is a regular 3x3 group of square (d = 40 cm) driven piles with lengths of 11,0 m. Spacing and pile-cap thickness are shown on Figure 1. Soil above the cap base is not included in the calculations and its influence is considered as a distributed load, equivalent to the overburden pressure at depth of 1,50 m ($q' = D_f.\gamma$ = 28,8 kPa).

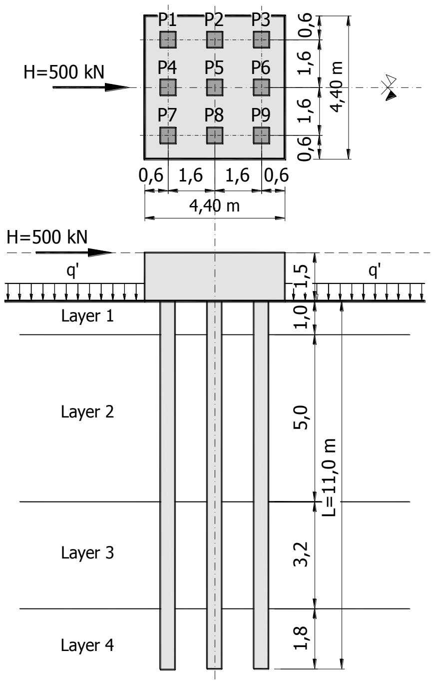

Figure 1. Pile frame geometry and soil layers

For all calculation models the center of the pile group is assumed at the origin of the global coordinate system (axis orientation differs, depending on software used).

Ground conditions are represented by four different soil layers and their geotechnical properties are given in Table 1. Layers 1-3 are cohesive clays, and Layer 4 is medium density sand.

The material for the piles and the pile cap is reinforced concrete (C25/30). Elastic isotropic behavior for the concrete is assumed with mean elastic modulus $E = 3.10^7$ kPa and ν = 0,2.

Table 1. Geotechnical parameters for soil layers

	k_h	E	ν	γ	φ'	c'
Units	*kN/m^4*	*MPa*	-	*kN/m^3*	°	*kPa*
Layer 1	2000	6,5	0,30	19,2	24°	7
Layer 2	2500	7,0	0,35	18,0	21°	25
Layer 3	3500	12,0	0,28	19,0	18°	30
Layer 4	8000	27,5	0,26	17,0	32°	3

3 ELASTIC SPRING MODELS

One of the first analytical solutions for piles is the analogy with beam on elastic foundation. Although this class of models is very simple and straightforward, the exact distribution of the stiffness with depth is unknown. The elastic bed gradient coefficients (k_h), specified in Table 1, vary in a wide range, so a number of solutions are performed with different values for k_h, averaged for depth of 8d (3,20 m).

3.1 Computational model and boundary conditions

A distinct characteristic of such models is the need to calculate equivalent spring stiffness for each node. In the case of pile frames it is common to account for group effects and force distribution between the piles. In this case it is done by introducing reduction of spring stiffness ($k_h^* = \eta.k_h$):

- for P1,P4,P7 – η_1 = 0,5;
- for P2,P5,P8 – η_2 = 0,7;
- for P3,P6,P9 – η_3 = 1,0;

Nodal spring stiffness is calculated using a simplified formula:

$$k_{x,i}\left(z_i\right) = k_{y,i}\left(z\right) = k_h^{*}.b_i.z_i.\overline{D},\ \ kN/m\ , \qquad (1)$$

where:

- k_h is the gradient of the lateral elastic bed coefficient, kN/m^4 (see Table 1). The values are from empirical relations given in [2], based on the consistency index (I_C);
- $\overline{D}$ – the effective width of the pile:

$$\overline{D} = 1,5d + 0,5\ m\,; \qquad (2)$$

- d is the width of the pile's cross-section;
- b_i are the length of the adjacent beam elements;
- z_i is the depth below the surface for the current node.

The vertical spring stiffness is assumed as:

$$k_{z,i}(z_i) = 2.k_{x,i}(z_i), \ kN/m \tag{3}$$

Alternatively coupled spring stiffness can be obtained using (4), but the difference in the results is not significant [3].

$$[k_{el}] = \int k(z).[\Phi]^T [\Phi] dz, \tag{4}$$

The deformed shape and discretization of the FEM model are shown on Figure 2.

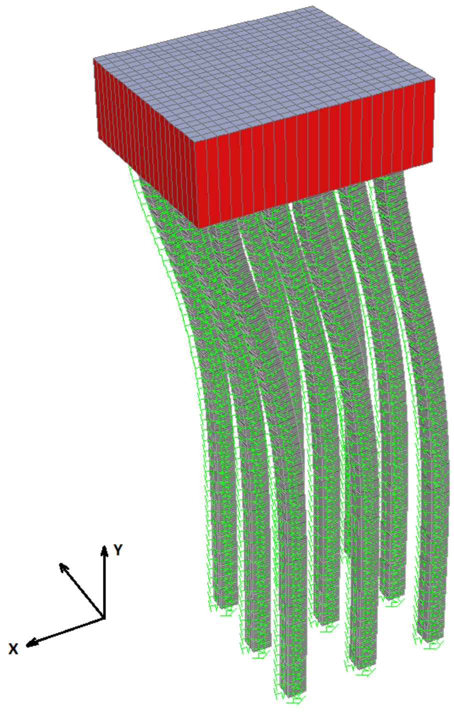

Figure 2. FEM model of the pile frame inside SAP2000

3.2 Results summary

The resulting deformed shape is shown on Figure 2. The data for displacements and forces is presented in Tables 2 – 4.

Table 2. Pile head displacements for different values of k_h

Stiffness, kN/m^4	$u_{x,top}$ *(for all piles)*	$u_{y,P1}$	$u_{y,P9}$
k_h = 1500	0,0093 m	0,0008 m	-0,0008 m
k_h = 2000	0,0077 m	0,0006 m	-0,0006 m
k_h = 2500	0,0066 m	0,0005 m	-0,0005 m

Table 2 indicates that for a 67% increase of spring stiffness, horizontal displacements decrease by 40%

Table 3. Maximum bending moments for different k_h values

Pile row	k_h= 1500	k_h = 2000	k_h = 2500
P1,P4,P7	83,9 kNm	80,1 kNm	77,0 kNm
P2,P5,P8	97,9 kNm	93,1 kNm	89,4 kNm
P3,P6,P9	114,3 kNm	108,4 kNm	103,9 kNm

Maximum bending moments vary in the range of 10%, which is relatively low and is attributed to the substantial difference in stiffness of the piles and the surrounding soil.

Table 4. Maximum shear forces (V_i) for different values of k_h

Pile row	k_h = 1500	k_h = 2000	k_h = 2500
P1,P4,P7	V_1 = 43,4 kN	V_1 = 43,4 kN	V_1 = 43,4 kN
P2,P5,P8	V_2 = 53,8 kN	V_2 = 53,8 kN	V_2 = 53,7 kN
P3,P6,P9	V_3 = 67,4 kN	V_3 = 67,2 kN	V_3 = 67,1 kN

The results in Table 4 clearly show that load distribution (η_1 = 43,4/67,4 = 0,64; η_2 = 0,80) differs from the input ratios η_i and that forces are rather proportional to the relative stiffness.

3.3 Comparison with results from the simplified design procedure

Bulgarian building code provisions [2] give a simple formula for calculating the horizontal displacement of a single pile, based on stiffness increase gradient k_h. Equivalence is assumed by taking only 1/9 of the total transverse force and no group effect is considered.

Results obtained by this procedure are presented in Table 5.

Table 5. FEM results compared to Code design formula

Stiffness, k_h	Code (BDS-EN)	FEM model
1500	u_x = 1,90 cm	u_x = 0,931 cm
2000	u_x = 1,60 cm	u_x = 0,767 cm
2500	u_x = 1,40 cm	u_x = 0,661 cm

4 2D PLANE STRESS MODELS

The two-dimensional half-space conditions are not very accurate for analyzing pile foundations. Nevertheless, this simplification is still used mainly because it can be performed using general purpose FEM software.

Two computational models are considered – linear elastic analysis in SAP2000 and an extended plastic analysis, using PLAXIS 2D.

4.1 Linear elastic model

This model uses two types of elements – membrane elements with thickness of 1,0 m for the soil volume and frame elements for the piles. The decision to model the piles as beams (neglecting their thickness) is mostly because of “shear-locking” of plane stress elements in bending.

To reduce the effect of boundary fixities on global results, the soil body extends on 35 m in horizontal direction and 25 m below the pile frame. Standard boundary conditions are applied and the loading is taken as $H_{2D} = 1/3H$, assuming equal load distribution.

The pile cap is represented with shell elements, and a rigid body constraint is applied for the corresponding nodes.

Results obtained from this analysis are represented by displacements of the pile cap (Table 6) and maximum forces at the connection of the piles with the cap (Table 7).

Table 6. Pile head displacements (at pile-cap base)

Pile row	u_x, *m*	u_y, *m*	θ_y, *rad*
(P1, P4, P7)	0,031	0,004	0,0019
(P2, P5, P8)	0,031	0,0	0,0019
(P3, P6, P9)	0,031	-0,004	0,0019

After analyzing the maximum shear forces V_i, alternative values for the distribution coefficients can be obtained. The factors are defined as:

$$\eta_i = \frac{V_i}{V_{i,\max}} . \tag{5}$$

Table 7. Distribution of forces in the piles (at pile-cap base)

Pile row	M_i ,*kNm*	V_i , *kN*	η_i , –
(P1, P4, P7)	87,4	51,3	1,00
(P2, P5, P8)	61,2	24,9	0,48
(P3, P6, P9)	87,4	51,2	1,00

4.2 Nonlinear plastic model (PLAXIS 2D)

As a better approximation to real soil behavior, the Mohr-Coulomb failure criterion is assumed for all soil layers with unassociative flow rule ($\psi = 0°$) [4]. Geometrical data is the same as in the linear elastic model and the piles are modeled with plate (frame) elements. Interface elements are defined on both sides of each pile (positive and negative), with the same parameters as the surrounding soil.

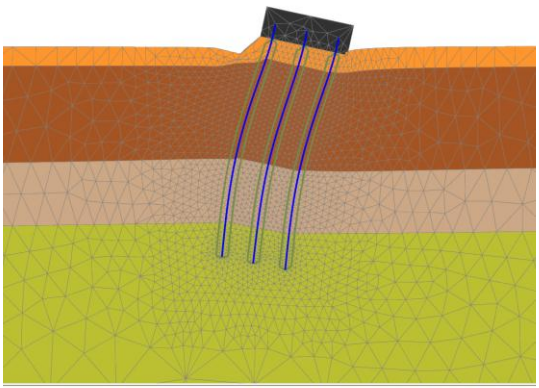

Figure 3. Deformed mesh around the frame, PLAXIS 2D

For a load of H = 500 kN, the horizontal displacement of the pile-cap is u_x = 0,035 m, which is close to the result obtained from the elastic solution, suggesting small plastic strains.

After the initial load step (for H = 500 kN), the transverse load is increased in three increments and the corresponding load-displacement curve is obtained (Figure 4). Displacements and forces for the increased loading are structurally unacceptable, but they provide information for the bearing capacity in terms of ULS. However, such extreme loading is only theoretical and actual failure mechanisms are not considered here.

Force distribution is presented in Table 8, and corresponding distribution factors can be calculated using (5).

Table 8. Distribution of horizontal forces in piles – F_h, kN

Load, H	P1,P4,P7 $F_{h,1}(V_1)$,	P2,P5,P8 $F_{h,2}(V_2)$,	P3,P6,P9 $F_{h,3}(V_3)$
500 kN	55,4	28,5	60,8
1000 kN	98,8	61,9	139,5
2000 kN	158,8	132,0	324,4

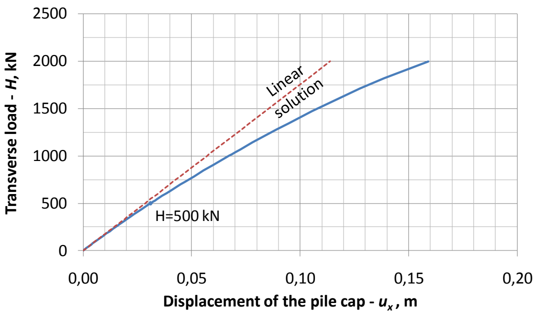

Figure 4. Load-displacement curve, PLAXIS 2D

5 3D MODELS

The out-of-plane effects and stiffness of the frame are essential for estimating its behavior. In order to get a better representation of the system, two three-dimensional models are evaluated.

Due to the symmetry in loading only half of the problem is analyzed using appropriate boundary conditions ($u_z = 0$ at the plane of symmetry; $F_{sym} = 0,5F$).

5.1 Linear elastic model (ANSYS)

In this model, piles are represented by solid (volumetric) finite elements and reduced integration is used (to avoid “shear-locking”). Elements SOLID187 are chosen (higher-order 10-noded tetrahedrons), which have quadratic displacement behavior. [5]

The deformed shape and displacement map for the pile frame are shown on Figure 5.

Forces in the piles are derived by means of differentiation of transverse displacements:

$$M_z(y) = \frac{d^2 u_x(y)}{dx^2} \cdot E_m I, \ kNm, \tag{6}$$

where:

- $u_x(y)$ is the transverse displacement result, mapped on the axis of the pile;
- $E_m = 3.10^7$ kN/m^2 and $I = 0,0256$ m^4.

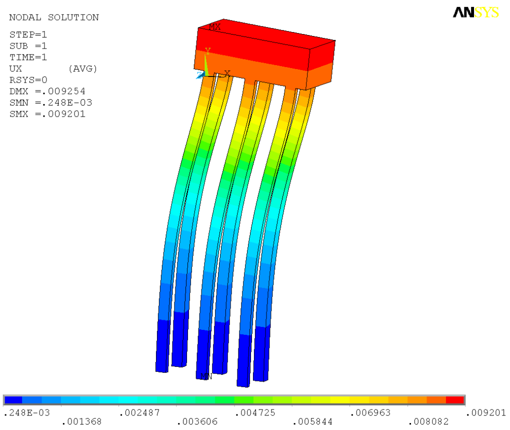

Figure 5. Displacements of the pile frame (isolated), ANSYS

5.2 Nonlinear plastic model (PLAXIS 3D)

Both 3D models share the same geometry, but here plastic behavior (MC failure criterion) is taken into account. The model is shown on Figure 6.

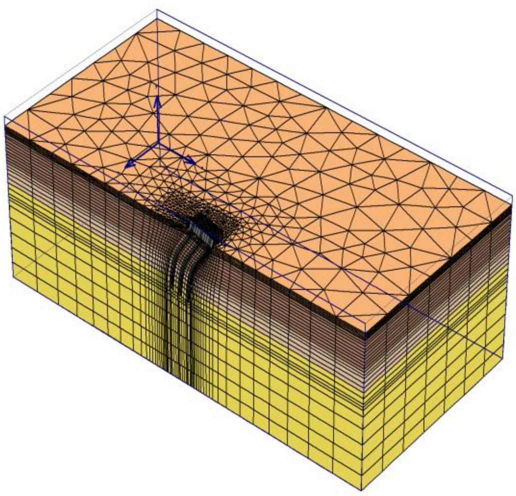

Figure 6. Computational model in PLAXIS 3D Foundation

5.3 *Summary of results for 3D models*

Displacements for the actual load of 500 kN are almost identical for the linear and nonlinear solutions, as observed in Table 9.

Table 9. Displacements of the pile-cap in the 3D models

Displacements	ANSYS 3D (Elastic)	PLAXIS 3D (Plastic)
u_x	0,0092 m	0,0099 m
$u_{y,min}$	-0,0009 m	-0,0011 m
$u_{y,max}$	0,0009 m	0,0011 m

Table 10. Distribution of shear forces in the piles

Pile shear forces, $F_{h,i}$ (V_i), kN	ANSYS 3D Elastic	PLAXIS 3D Plastic
P1 (=P7)	47,61 kN	48,69 kN
P2 (=P8)	38,37 kN	35,92 kN
P3 (=P9)	47,11 kN	49,30 kN
P4	34,96 kN	36,82 kN
P5	24,60 kN	27,14 kN
P6	35,00 kN	36,68 kN

Applying additional load steps (H = 2000 kN) yields the load displacement curve, shown on Figure 7. No apparent limit force can be determined (without extrapolation), although the decrease in stiffness is evident.

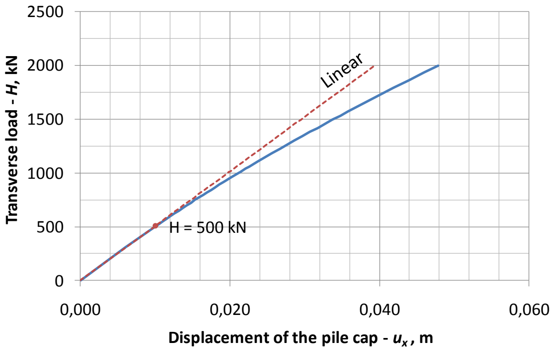

Figure 7. Load-displacement curve from PLAXIS 3D

6 CONCLUSIONS AND RECOMMENDATIONS

Summary of results for group effect factors (η_1) and horizontal displacements (u_x) from all analyses is presented in Table 11.

Table 11. Comparison of distribution for H = 500 kN.

Model	η_1	η_2	η_3	u_x, m
BG code	0,50	0,70	1,00	0,019
1D Springs	0,65	0,85	1,00	0,009
2D Elastic	1,00	0,48	1,00	0,031
2D Plastic	0,91	0,47	1,00	0,035
3D Elastic	1,00	0,78	0,99	0,009
3D Plastic	0,98	0,73	1,00	0,010

For practical cases of transversely loaded piles, linear-elastic behavior is an acceptable approximation. However plane stress solutions overestimate displacements with factors of around 3,0. From designers' point of view this is "on the safe side", but their application should be avoided.

Elastic spring models are efficient for soil-structure interaction, but only after proper spring stiffness distribution from more detailed 3D models is obtained. Equivalent elastic bed stiffness, as function over the length of each pile, can be obtained using the formula:

$$k_x(z) = \frac{d^4 u_x(z)}{dz^4} \cdot \frac{E.I}{u_x(z)}, \quad kN/m^2, \qquad (7)$$

where:

- $u_x(z)$ is obtained from a 3D model and is numerically differentiated
- $E.I$ is the flexural stiffness of the pile.

Pile cap fixity affects results significantly and in case of soil-structure interaction, the partial restraint should be taken into account in the 3D model as an equivalent fixity.

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