3. Stereoscopic vision 2 hours

Aim:
Understanding and application of stereoscopic vision – not only for photogrammetry –

Theory:
general methods of stereoscopic vision
measurement

3.1. Principles of binocular vision

The main property of binocular vision is the ability of depth perception. Nevertheless there is a property of monocular vision that allows estimating the distance to the objects. It is based on the focusing accommodation of the individual eye. Monocular vision has two important for the measurements properties. These are angles of distinct sight. There are of two types. First of it is the ability to distinguish two luminous points. It is at about 45”. Second type ability for distinct sight is for possibility to distinguish two parallel lines. It is estimated to be about 20”.

Binocular vision is very important for acceptance of space distribution of objects under observation. In stereovision everyone put his eyes in such manner that central rays of sights intersect over the object of interest. The point of intersection is called fixing point of binocular vision. The images of fixing point lie in the central axillae of the eyes. The distance between the front nodes of the eyes is termed eye base (or eye spacing) b_e. The length of this base is between 58 up to 72 mm and normally is about 65mm. The angle between rays to the same point is termed parallax angle γ. The angle between rays for fixing point is called convergent angle. The values of these angles are small.

$$\gamma = \frac{b_e}{D} \tag{3.1}$$

For the distance of better viewing (25cm) this angle is about 15°. The ability of human vision to accept the images in both eyes as one is very important. It is possible only for points that lie near to central point of the eye. The points in the eyes lie that are at the same distance from the central point are symmetrical. The images of objects over the symmetrical points are assumed at the same distance as the distance for the fixing point. If the distances (angle distances) are different the points are accepted as near or far from the distance to the fixing point.

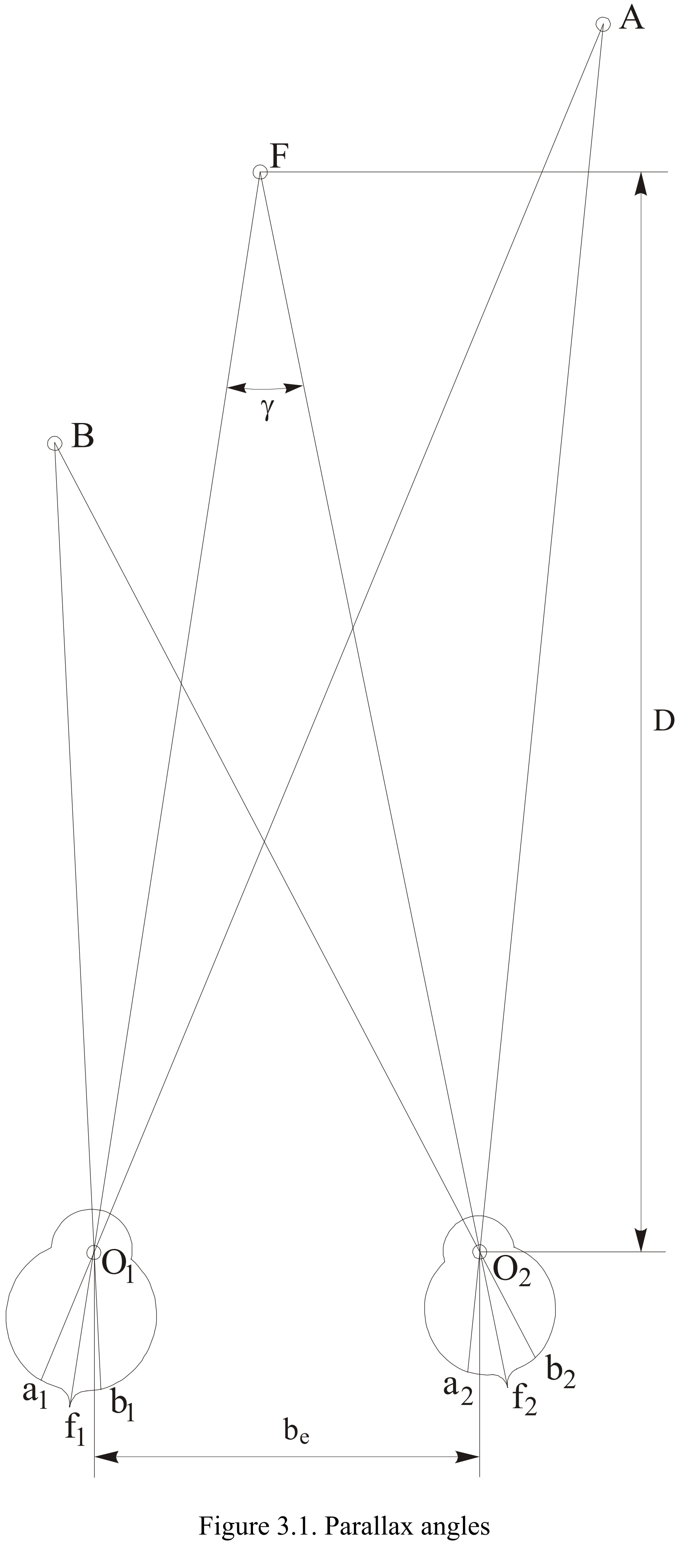


Figure 3.1. Parallax angles

When the difference in angle distance for two points is larger they are accepted as different and the observed image is forked. This is measured only for points which parallax angle differs less then 70' (positive or negative) then parallax angle for fixing point.

$$|\gamma - \gamma_F| \leq 70' \qquad (3.2)$$

For the accuracy of depth measurement and estimation it is important he minimal value of difference of parallax angle for two points at different distances for which the distance is accepted as not the same. This angle is estimated to be about $\Delta\gamma_1 = 30''$.

The distance for the parallax angle equal to the $\Delta\gamma_1$ is termed the radius (distance) of unarmed binocular vision. It is estimated to e about 450m. (see Appendix 3.1).

This distance can be enlarged by magnification of viewing system or by the enlargement of the base of entrance objectives.

3.2. Methods of stereoscopic observation

Stereo effect is possible to obtain not only viewing the objects at the different distances but observing projective images of objects. In this case the distance between images (photos) must be the same as the eye spacing. By this reason the size of the photos could not be too large (approximately 60 mm). In the process of observation between two eyes must put separating plate. This ensures observation the corresponding image with each of the eyes.

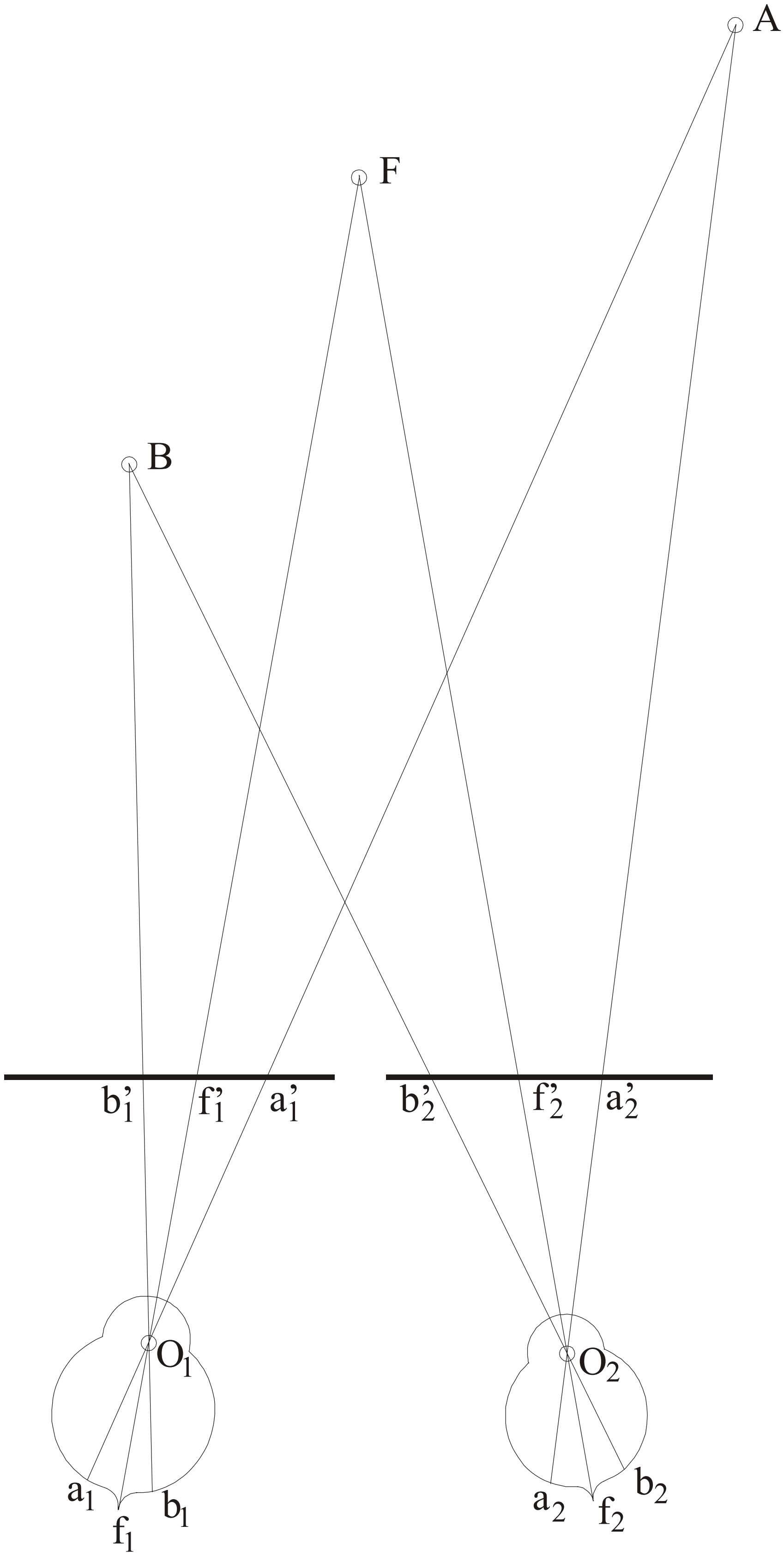


Figure 3.2. Stereoscopic observation without equipment

To obtain good stereo effect it is necessary to satisfy several conditions.

1. Photos must be produced from different projection centers;
2. Scale differences must be less than 16%
3. Every eye must see its own separate photos;
4. Photos must be disposed in such way that rays of view to intersect
5. The angles of intersection must be less than 16°;
6. For the used values of parallax angles the eye accommodation must be possible.

Better results can be reached if stereoscopic equipment is used.

Several transformations in the dimensions of the perceived model may happen. If the height and base are same as in the three dimensions they is no change of scale and height. But this case is ideal and practically not happens. The observed height and scale depends on base/height ratio at the moment of registration and the base distance ratio at the moment of observation. This is graphically shown on the following figures. The next figure shows the registration of the image from normal angle and wide angle camera from different distances (flight heights).

Registration

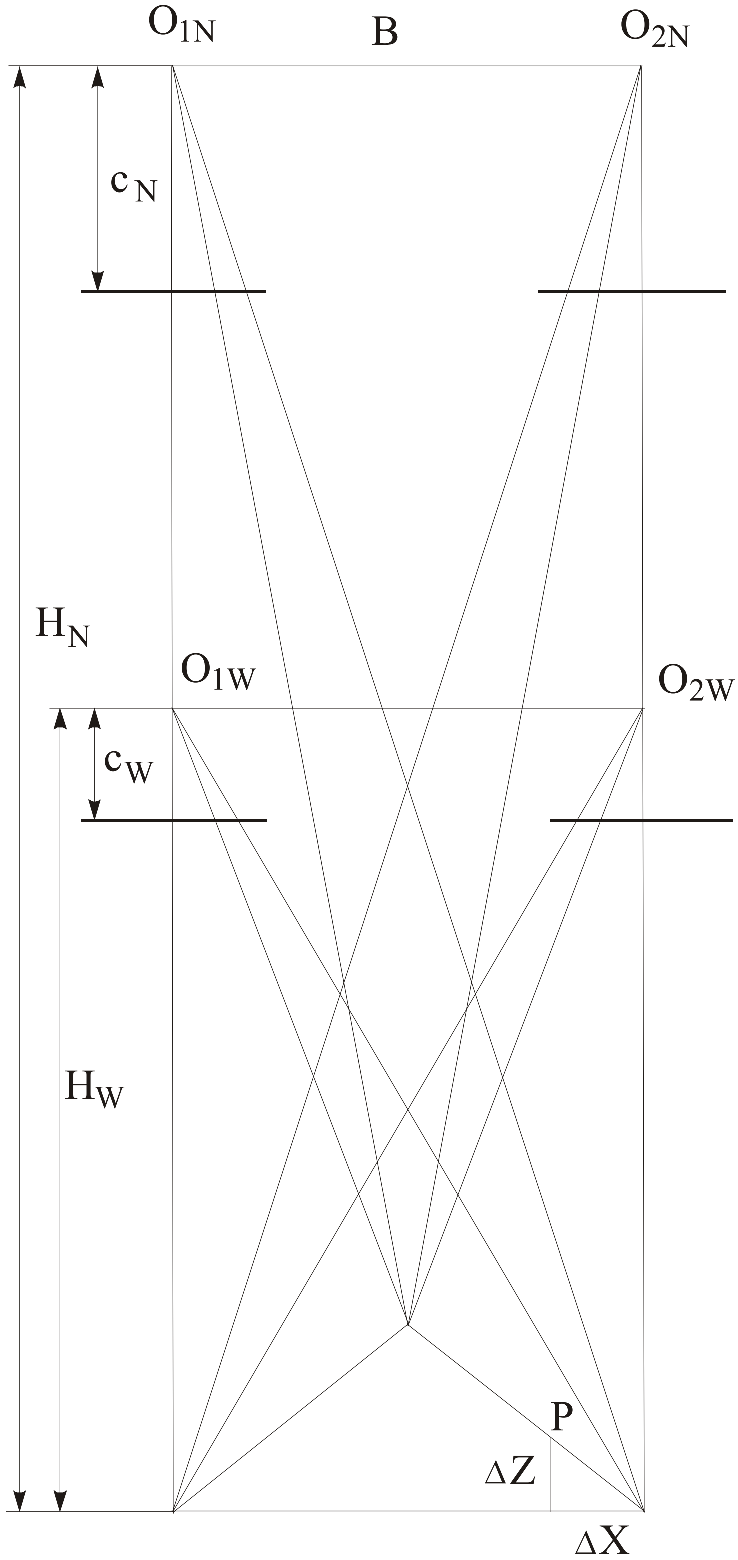
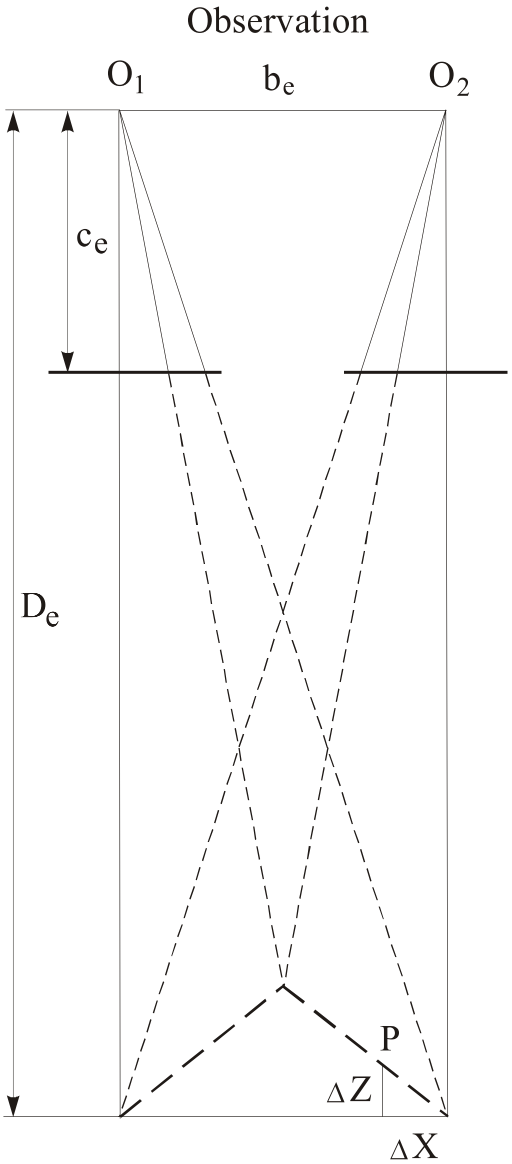
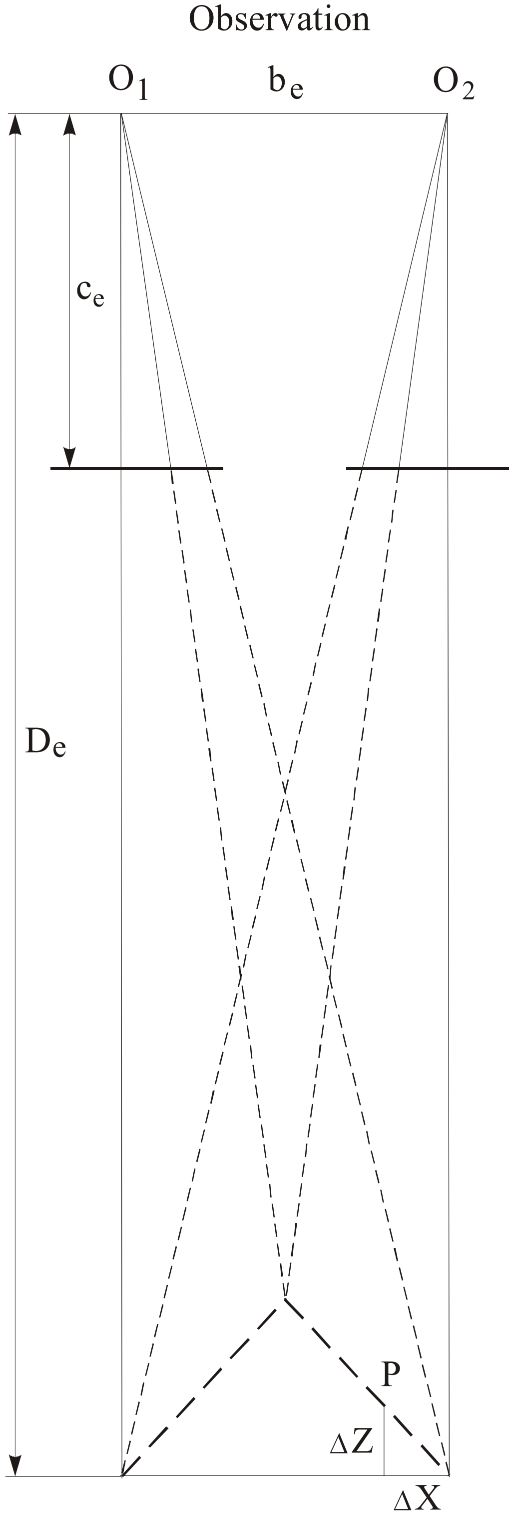


Figure 3.3. Photo capturing

The images from normal angle camera observed with viewing system with same B/D ratio and with shortest D_e.



Normal angle camera's photos
Normal image distance D_e



Normal angle camera's photos
Long image distance D_e

Figure 3.4. Observation of normal angle photos

The vertical exaggeration is ratio between base/height of stereo photos and base (eye spacing)/distance ratio in the observation system.

$$e_Z = \frac{B}{H} : \frac{b_e}{D_e} \tag{3.3}$$

For wide angle cameras the results are analogues. (see Appendix 3.2). The table below shows the vertical exaggeration for viewing system with be/De=65:400=1:6.2. The results for same narrow angle, normal and wide angle cameras follows [Kraus K., 1993, Photogrammetry v.1] are presented in the table bellow.

Table 3.1

	Narrow angle	Normal angle	Wide angle	Super-wide angle
C [mm]	600	300	150	85
B/H	1:6.6	1:3.3	1:1.6	1:0.9
e_Z	0.9	1.9	3.8	6.7

The exaggeration of the relief in stereo model is important for photo-interpretation. For photogrammetry it is not of such great importance but it is meaningful for exact identification of stereo points during measurements.

For practical purposes are applied mirror stereoscopes, lens stereoscopes and lens mirror stereoscopes. Mirror stereoscope only changes the effective displacement between two photos.

Lens stereoscope produces enlargement of images. The scale of enlargement could be simple defined by the following figure

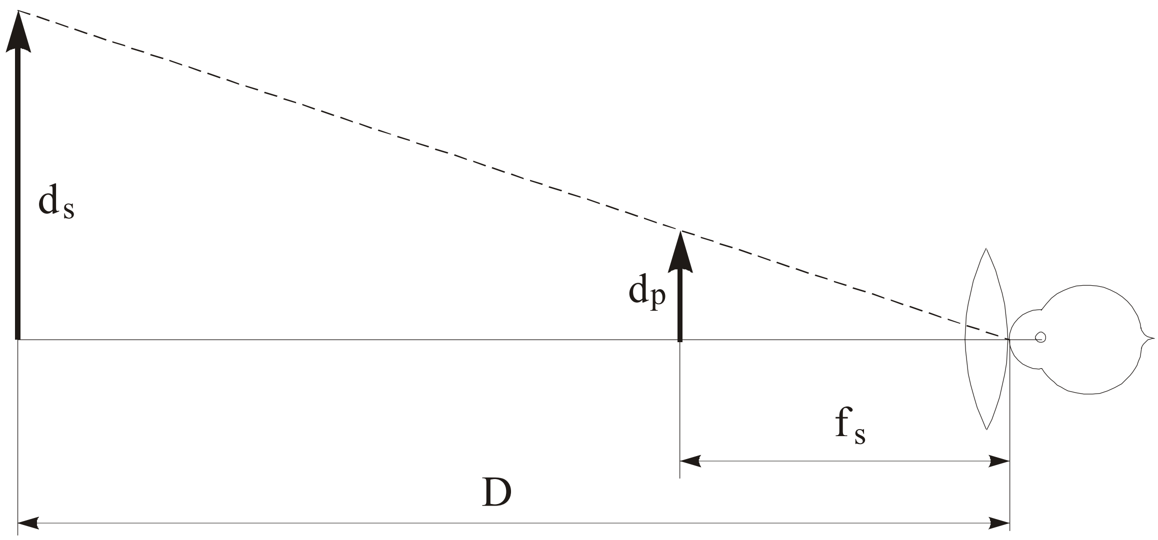


Figure 3.5. Magnification of lens stereoscope

The lens enlarges the effective angle of viewing the objects. This allows moving the viewing object closer to the eye and the eye can still focus. The magnification ratio depends on focusing distanced of the lens f_S and distance of best viewing D.

$$M = \frac{d_s}{d_p} = \frac{D}{f_s} \tag{3.4}$$

The lens stereoscope magnifies the apparent parallax p and base b thus there is the enlargement of the whole model. This does not change the observed ratio h/b and respectively the vertical exaggeration. The derivation of enlargement ratio is shown in Appendix 3.3. It is presented by equation

$$k = \frac{M}{s} \tag{3.5}$$

where *s* is the scale of the photograph (image).

The simple lens mirror stereoscope is similar to lens stereoscope but with addition of two mirrors or prisms in the path from left and right photos. This increases the separation between two photos so the larger photographs could be observed without overlapping at the center of stereoscope working plane.

The separation of stereoscopic pairs is selected empirically by most observers. It is chosen by the best stereoscopic viewing of the objects.

From the relation (3.4) it is found the base-distance ratio for stereoscope

$$\frac{D_e}{b_e} = e_z \frac{H}{B} \tag{3.6}$$

The ratio distance/base for lens stereoscope is connected with displacement w with relation.

$$\frac{D_e}{b_e} = \frac{f_s}{b_e - w} \tag{3.7}$$

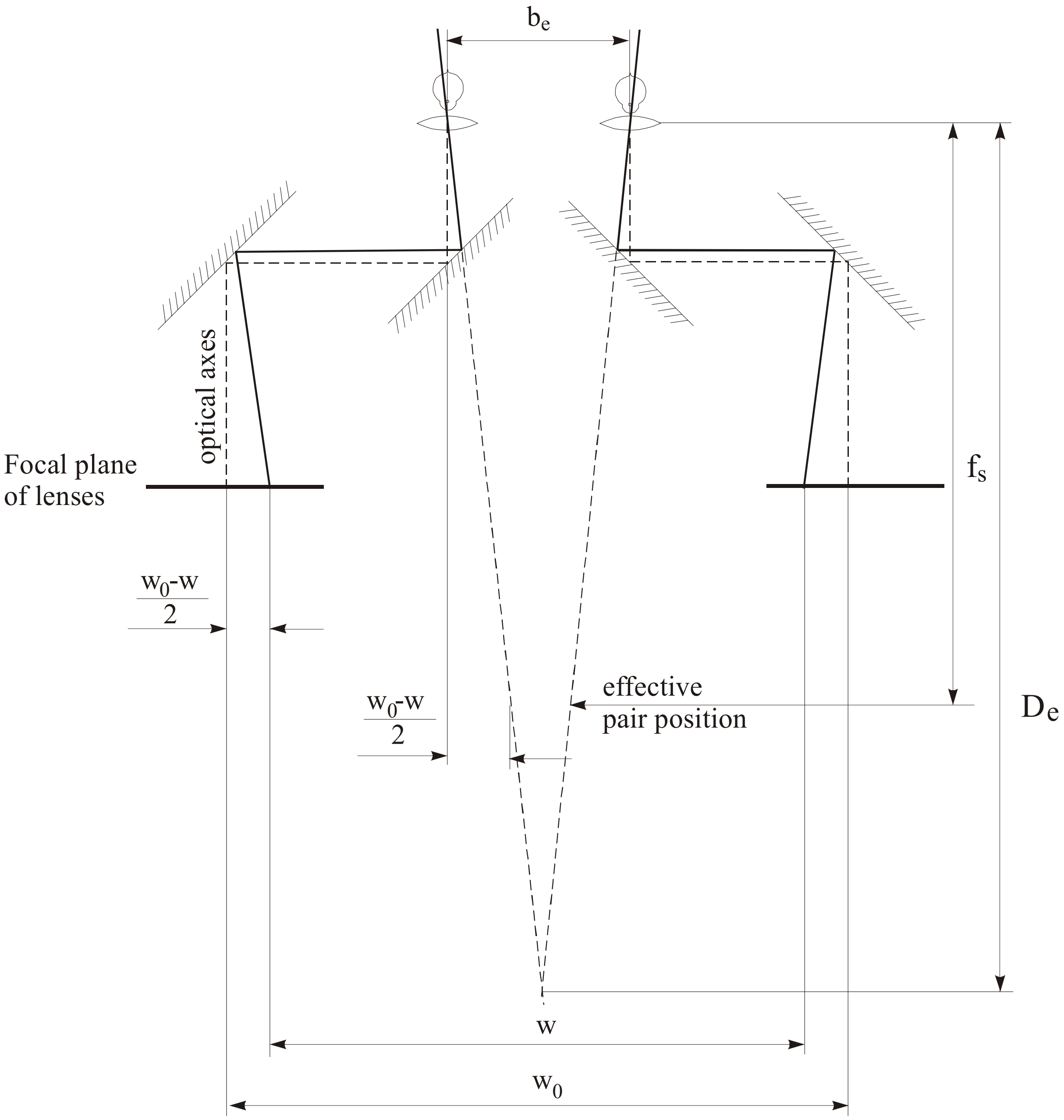


Figure 3.6. Mirror stereoscope geometry

The relation for the optimal displacement of mirror stereoscope is similar

$$w = w_0 - \frac{f_s.B}{e_Z.H} \tag{3.8}$$

where w_0 is the base defined by the mirrors.

3.3. Methods of stereoscopic measurement

3.3.1 Observation and measurement of photo images

Parallax bar measurements

Stereoscopic measurement may be realized by parallax bar that has the measuring marks. Initially we read the micrometer screw at principal point of one of the photos. For this point we know the real value of parallax p_ξ. The image base b_0 can be measured by the distance between the images of the objects at the principles points of the photos of the stereo pair. For this point the actual value of height must be known. If it not possible the usage of any other point is possible. The reduction of measured micrometer values is made respectively to this point.

The figure for height differences is shown below.

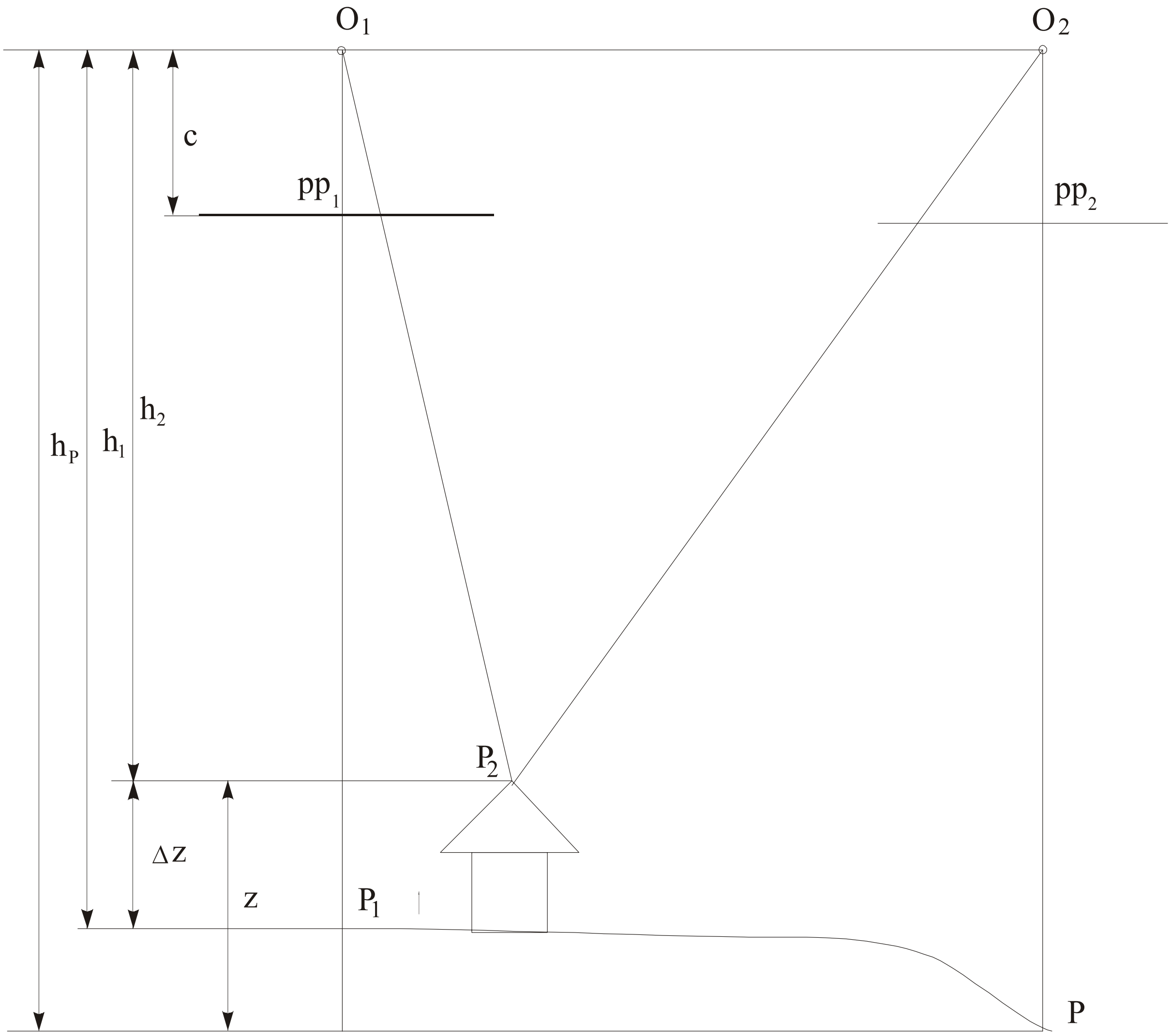


Figure 3.7. Parallax bar measuring parameters

$$\begin{aligned} \Delta p_{\xi,i} &= p_{\xi,i} - b_P = n_i - n_P = (n_i - n_A) + (n_A - n_P) = \Delta p_{\xi,iA} + \Delta p_{\xi,AP} \\ \Delta p_{\xi,iA} &= p_{\xi,i} - (\Delta p_{\xi,AP} + b_P) \end{aligned} \tag{3.9}$$

For height difference between points *A* and *i* can be written the following expression

$$z_i = \frac{h_P}{b_p + \Delta p_{\xi,iP}} \Delta p_{\xi,iP} \tag{3.10}$$

The derivation of the relation is shown in Appendix 1.5

Floating mark principle

Another method of stereoscopic measurement is known as **floating dot principle**. Two lightning point or dots are introduced in the plane of stereoscopic pair of photos. They can be aligned with any two corresponding points on the pair. The parallax p_ξ produces a disparity angle α, which is perceived by the observer as the height h. The measuring is made by virtual mark hat is realized y two dot or light marks exposed at the center of the viewing field (placed on the path of central rays) for the two photos of stereo couple. By moving the position of image respectively to the position of flying point mark the perception of moving up and down of mark is obtained.

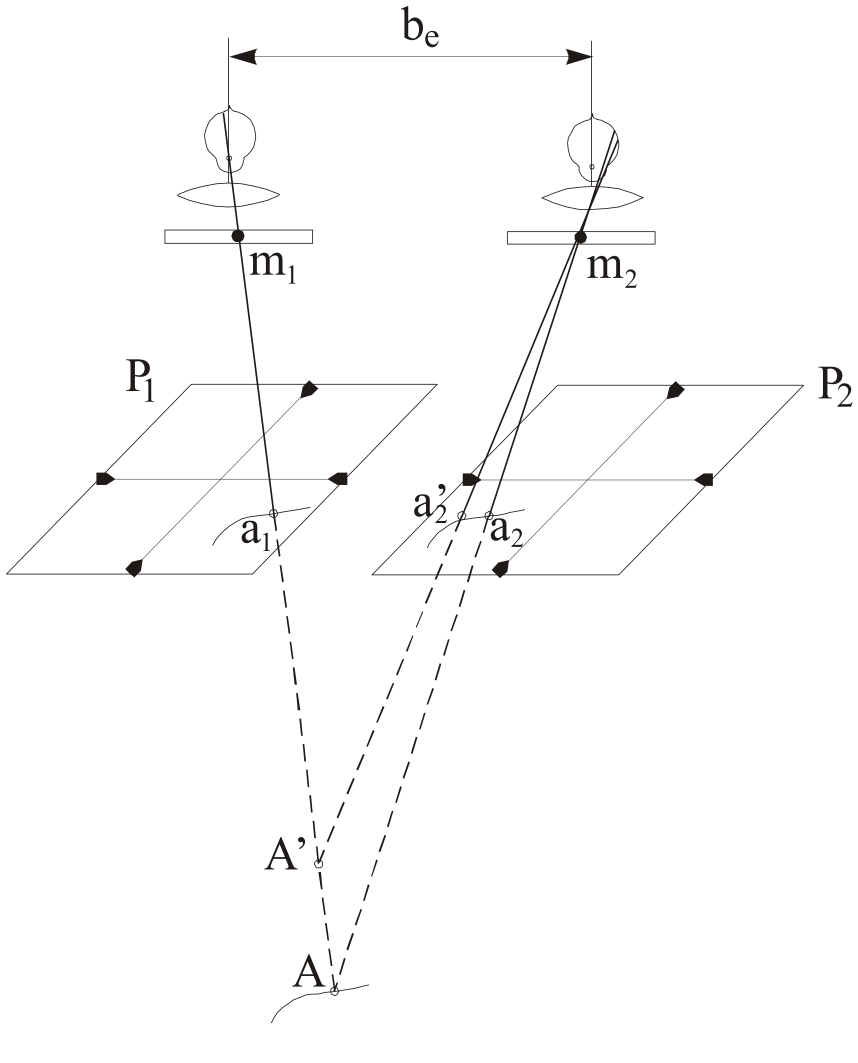


Figure 3.8. Floating mark principle

Real mark and anaglyph vision

Another principle that is used in analogue apparatus is the principle of real mark. The model is created in model space and is observed by the operator with anaglyph glasses. Two images are produced by color filterers and are observed trough the glasses with the corresponding color.
The image is visualized over the flat screen. On it there is a lightning point that is the real mark. When the position in plane and height coincides with point in the model the perception of coincidence between model and measuring point is reached.
The anaglyph principle is used to observe the printed stereo mages. The difference in this case is that for printed images the visible image is in the opposite color. For example eye with the green glass will see the red image and the eye with the red glass will see green image. The reason for this is that the white background is visible as green through green glass and the green image is not seen on it. The red glass passes red light from the white background and the red image is not seen on it (through red glass).

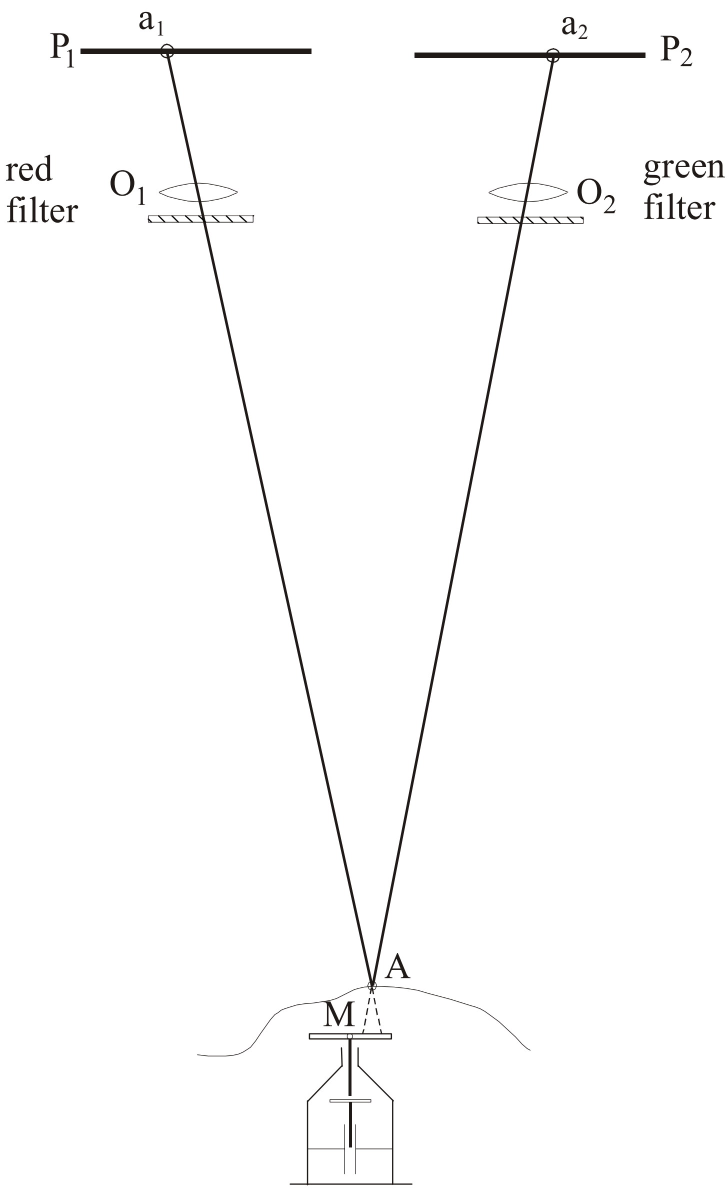


Figure 3.9. Real mark observation

3.3.2. Stereo measurements of digital images

In digital photogrammetric systems are used some of methods that are applied in analogue and analytical photogrammetry. But the source of image in all cases is screen of the monitor. There are used four types of system for stereo observation and measurement.

1. **Stereoscopic viewer with split screen**. The screen of the monitor is separated into two parts vertically and in each part is visualized one of the stereo images. The mirror stereoscope is situated in front of the screen. The system does not require the expensive parts. The parameters of stereoscope are adopted for screen size of the monitor. This is the main limitation in this system. There is a possibility for monoscopic and stereoscopic observation of the stereo pair.

2. **Anaglyph observation of stereo images**. In this case the monochromatic images are colored in two colors and overlapped over the screen. The main disadvantage is that such approach is suitable only for monochromatic images. The overlapping color vector graphic if exist is not seen well. This made system very poor applicable for plotting of complex areas and colour coded boundaries.

The third and fourth systems are based on **usage of polarized light**. The system that are used are of two types: a) with passive screen and active spectacles; b) active screen and passive spectacles. The main principle of operation is based on the passing of the polarized light through the spectacles with polarized glasses. This switching of the images comes more than 50 times per second and due to the features of human eyes to memorize images for the short time the visible image is stable. The plane of polarization is changed so the corresponding glass blocks or passes light. The main disadvantage of this system is their high price.

3. **Polarised light observing system with active screen**. Active screen is mounted in front of the monitor screen. Its plane of polarization is changed by electrical field. The passive spectacles, that are used, have glasses with perpendicular planes of polarization. When the plane of the polarization of screen and glass are the same the image is visible. So at same time the screen is visible only through one of the glasses. The change of the left and right image is made synchronously with the change of polarization.

The principle diagram of this method is shown on the figure below.

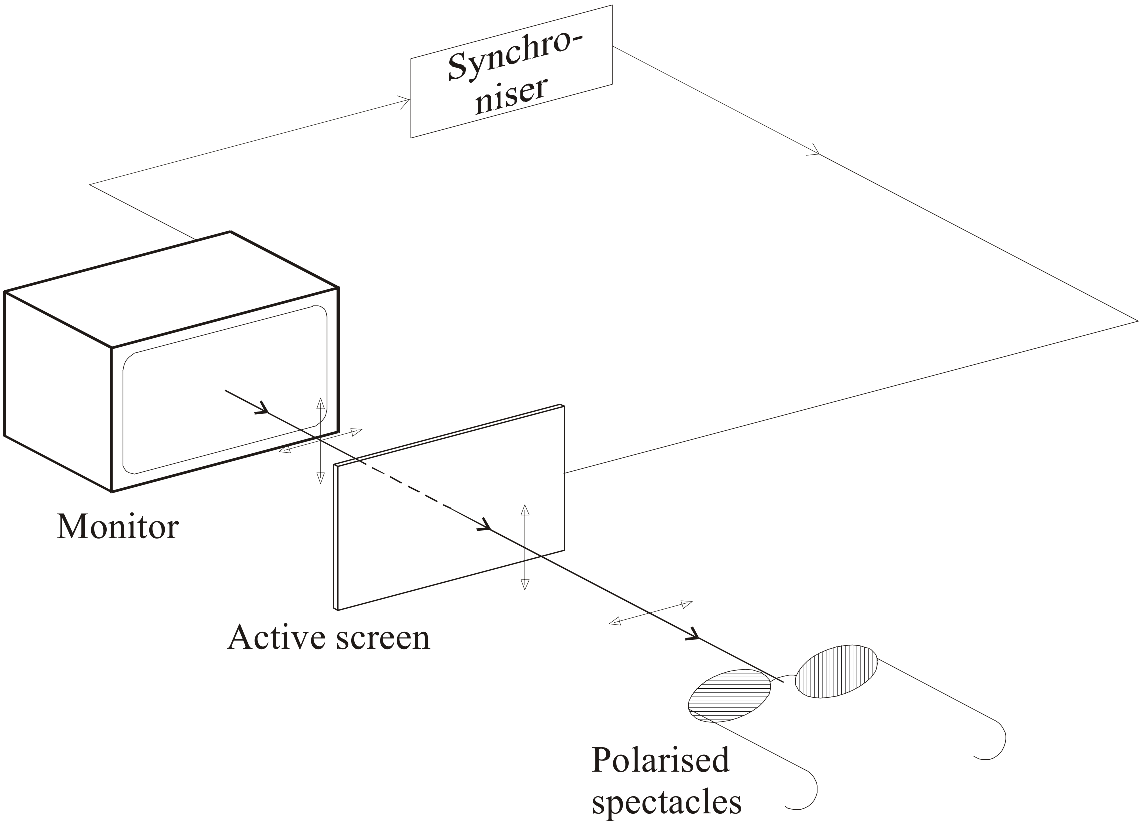


Figure 3.10. Observing stereo system with active screen

4. **Polarised light observing system with active spectacles.** This system does not uses active screen. The commutation of images is in the spectacles. Every one of their glass consists of two parts – fixed polarised glass and active glass electrically controlled. The synchronization between monitor and spectacles is ensured by infrared or ultra-sonic emitter situated over or near to monitor and receiver in the spectacles. Main disadvantage of this system is that their glasses are too heavy and expensive. The development of technology makes this disadvantage more and more unimportant. This system has more advantages and is widely used.
The principle diagram of this method is shown in e following figure.

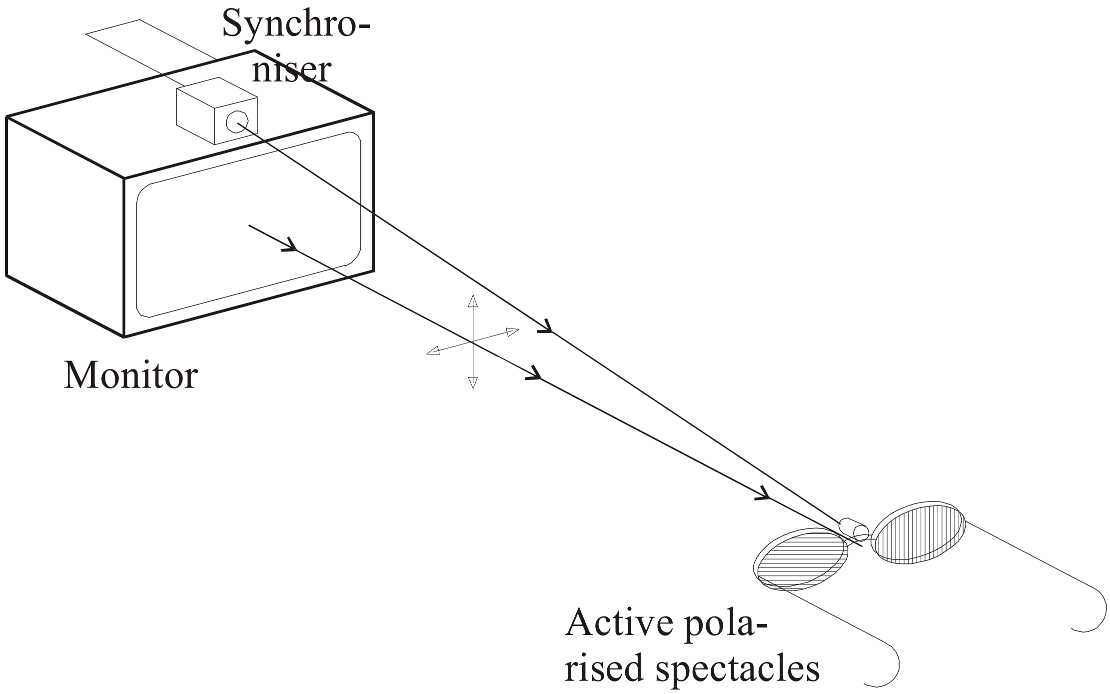


Figure 3.11. Polarised light method with active spectacles

The comparison of the main properties of the four methods is shown in the following table.

Table 3.2

Property	Stereoscopic viewer	Anaglyph method	Active polarized screen	Active polarized spectacles
Images	Color	Monochromatic	Color	Color
Number of observers	1	3-4	2-3	2-3
Monitor	Standard	Standard	High frequency	High frequency
Cost	Low	Medium	High	High
Main disadvantage	One observer	Monochromatic images	Low image intensity	Heavy spectacles

In the recent ears the most perspective method for stereo measurement in digital photogrammetry is usage of systems with active spectacles.

Appendixes

Appendix 3.1

The accuracy of depth estimation for unarmed vision we may obtain from the equation for parallax angle at optimal distance.

$$D = \frac{b_e}{\gamma} \tag{3.3}$$

Differentiating this we obtain

$$\Delta D = -\frac{b_e}{\gamma^2}.\Delta\gamma = -\frac{b_e}{\left(b_e / D\right)^2}.\Delta\gamma = -D^2\frac{\Delta\gamma}{b_e} \tag{3.4}$$

Analogue it is defined the distinct ability to distinguish two vertical lines. Empirically it is estimated to be about $\Delta\gamma_2 = 10"$.

The distance for the parallax angle that is equal to the $\Delta\gamma_1$ is termed the radius (distance) of unarmed binocular vision. It is estimated to be about

$$R_b = \frac{b_e}{\Delta\gamma_1} = \frac{65.10^{-3}\rho"}{30"} = 450m \tag{3.5}$$

where $\rho" = \frac{180.3600}{\pi}$

Practically this distance is little bit more. This distance could be enlarged artificially if we use binoculars or stereo trench-periscope (stereo viewing tube).

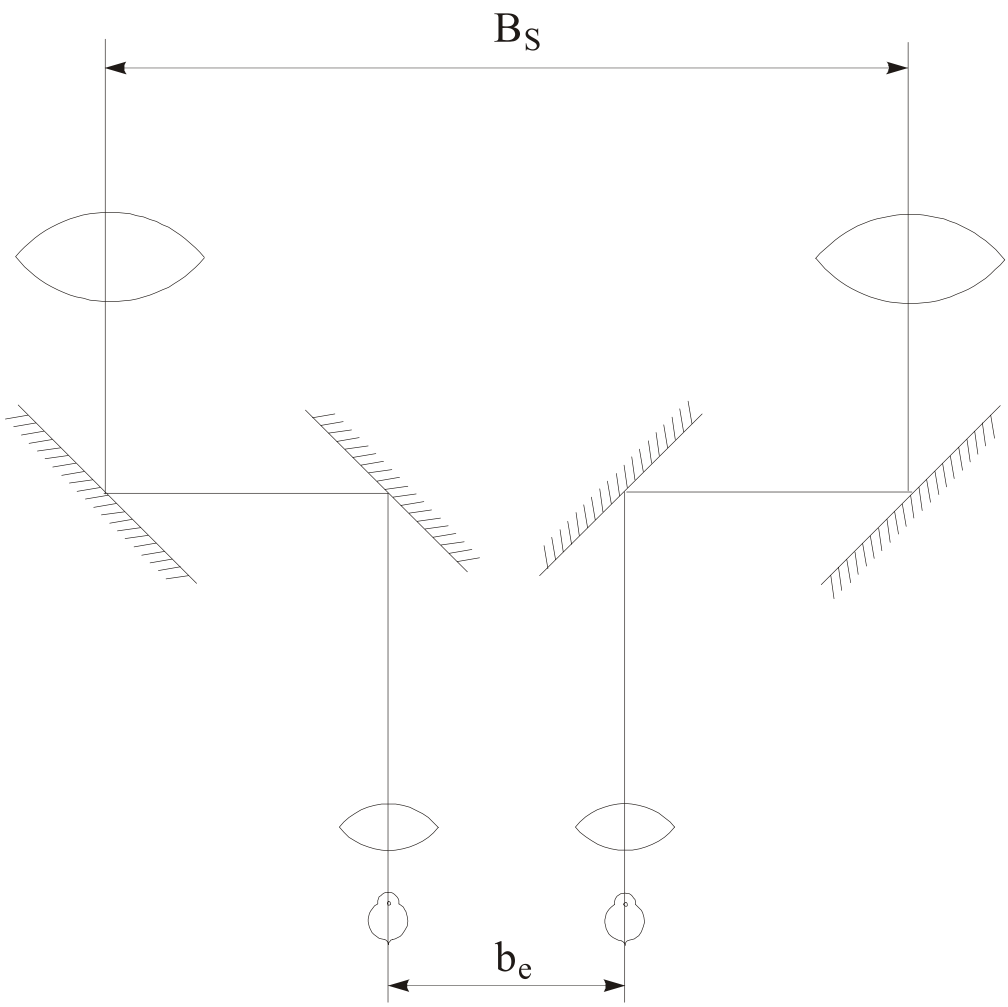


Figure 3.12. Stereo viewing tube

Two factors enlarge this radius: magnification of the images and the bigger base then eye spacing.

$$R_a = \frac{B_s}{b_e} v R_b = \omega . R_b \tag{3.6}$$

where $\omega = \frac{B_s}{b_e} v$

Appendix 3.2

The planimetric to height enlargement

There are discussed the observation of images taken with wide-angle camera with the viewing system for normal photos and with short distance. The usage of appropriate short distance ensures the same planimetric scale to height ratio as in the object. This is not possible for all camera types.

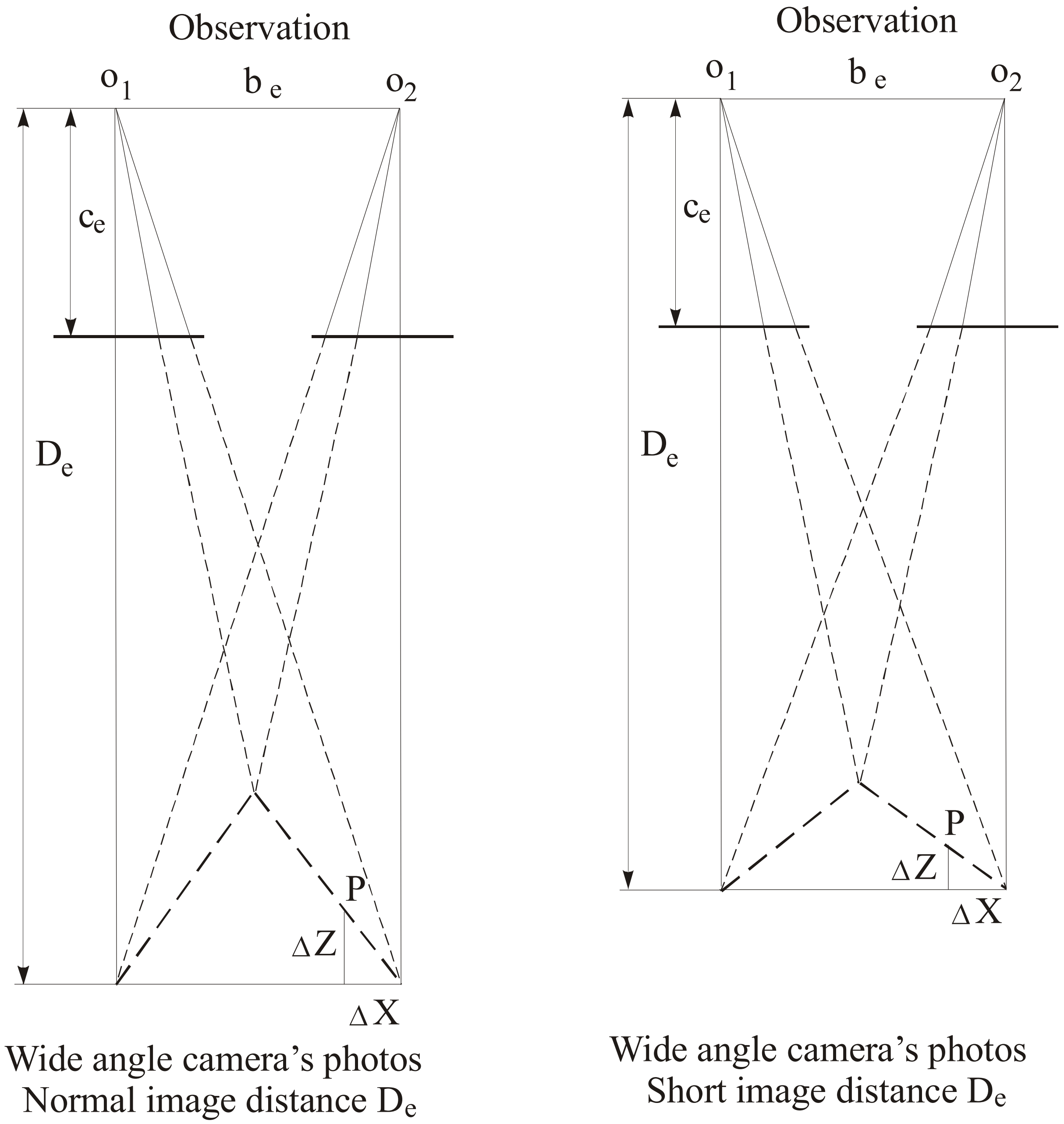


Figure 3.13. Observation of wide angle photos

The estimation of vertical exaggeration e_Z is evaluated from the figures 3.5 and 3.6.

$$e_z = \frac{\Delta Z_e}{\Delta X_e} : \frac{\Delta Z}{\Delta X} \tag{3.7}$$

The X displacement is chosen as the n-th part of base distances.

$$\Delta X = \frac{B}{n} \quad \Delta X_e = \frac{b_e}{n} \tag{3.8}$$

The values of ΔZ is estimated from relation between parallax and projection center distance

$$\Delta Z0 = H_U - H_P = \frac{c.B}{p_{\xi,U}} - \frac{c.B}{p_{\xi,P}} \tag{3.9}$$

The horizontal parallax for plane U exists in the observation too.

$$D_e = \frac{c_e b_e}{p_{\xi,U}} \quad \Rightarrow \; c_e = \frac{D_e p_{\xi,U}}{b_e} \tag{3.10}$$

By analogy we obtain for ΔZ_e the relation

$$\Delta Z_e = c_e b_e \frac{p_{\xi,P} - p_{\xi,U}}{p_{\xi,U} \cdot p_{\xi,P}} = \frac{D_e p_{\xi,U} b_e}{b_e} . \frac{p_{\xi,P} - p_{\xi,U}}{p_{\xi,U} \cdot p_{\xi,P}} = D_e \frac{p_{\xi,P} - p_{\xi,U}}{p_{\xi,U} \cdot p_{\xi,P}} \tag{3.11}$$

Substituting the values for ΔZ and ΔZ_e in relation for e_Z we finally obtain

$$e_Z = \frac{D_e p_{\xi,U}}{b_e c} = \frac{D_e}{b_e} . \frac{B}{H} = \frac{B}{H} : \frac{b_e}{D_e} \tag{3.12}$$

The result shows that vertical exaggeration is ratio between base/height of stereo photos and base (eye spacing)/distance ratio in the observation system.

Appendix 3.3

The following relation defines the magnification of the object

$$k = \frac{d_s}{d_t} \tag{3.14}$$

where d_s is visible size and d_t is size at the object space

The scale of photograph *s* is defines as

$$s = \frac{d_t}{d_p} \tag{3.15}$$

For equation of lens magnification

$$d_s = \frac{D}{f_s} d_p \tag{3.16}$$

Solving the above equations gives the object magnification

$$k = \frac{D}{f_s} . \frac{d_p}{d_t} = \frac{D}{s.f_s} = \frac{M}{s} \tag{3.17}$$

Appendix 3.4

The optimal displacement for mirror stereoscope is depending on the displacement due to the mirrors (from similar triangles) on the figure 3.6.

Solving this and substituting the value from ratio equation we obtain

$$w = b_e - f_s : \frac{D_e}{b_e} = b_e - f_s : \frac{D_e}{b_e} = b_e - \frac{f_s.B}{e_Z.H} \tag{3.18}$$

For mirror stereoscope the relation is slightly different (from figure 3.8)

$$\frac{D_e}{b_e} = \frac{f_s}{w_0 - w} \tag{3.19}$$

For the optimal displacement we obtain

$$w = w_0 - f_s : \frac{D_e}{b_e} = w_0 - f_s : e_Z \frac{H}{B} = w_0 - \frac{f_s.B}{e_Z.H} \tag{3.20}$$

Appendix 3.5

Observer is using stereoscope (more often mirror lens) and is coincides the marks with the stereo object. At this moment the value read from the drum scale is fixed. The measured values are relatively to some base point for which the real parallax must be known.

$$p_{\xi,i} = p_{\xi,0} + (n_i - n_0) \tag{3.21}$$

The difference of the two readings of the micrometric screw, relatively to any arbitrary origin, is the parallax difference.

$$n_2 - n_1 = \Delta p_\xi = p_{\xi,2} - p_{\xi,1} \tag{3.22}$$

The heights h1 and h2 can be derived for the normal case

$$h_1 = \frac{c.B}{p_{\xi,1}} \quad h_2 = \frac{c.B}{p_{\xi,2}} \tag{3.23}$$

For height difference we obtain

$$\Delta z = h_1 - h_2 = \frac{c.B}{p_{\xi,2}} - \frac{c.B}{p_{\xi,1}} = c.B \frac{p_{\xi,2} - p_{\xi,1}}{p_{\xi,1} \cdot p_{\xi,2}} = c.B \frac{\Delta p_\xi}{p_{\xi,1} \cdot p_{\xi,2}} \tag{3.24}$$

After substitution

$$h_1 = \frac{c.B}{p_{\xi,1}} \quad \text{and} \quad p_{\xi,2} = p_{\xi,1} + \Delta p_\xi \tag{3.25}$$

The expression for height difference could be presented in the following form

$$\Delta z = \frac{h_1}{p_{\xi,1} + \Delta p_\xi} \Delta p_\xi \qquad (3.26)$$

In case when the first point is base point we obtain

$$z_i = \frac{h_0}{b_e + \Delta p_{\xi i}} \Delta p_{\xi i} \qquad (3.27)$$