Concept Hell/Pfeiffer Tuesday, 14. February 2003

5. Structure of plotting methods 2 hours

Aim:
Detailed knowledge for possible applications

Theory:
Structure of plotting methods
Application and products of the different methods
Difference between stereoscopic and point wise plotting

5.1. Plotting methods

The data captured by photogrammetric equipment may be recorded in different way. They exist three main forms of data registration.

- set of points;
- lines data;
- image data.

From mathematical point of view to these three types of data correspond points, lines, surfaces.
Each type of data could be presented in analogue or digital form.
Set of points for example could be registered on sheets (paper, plastic, metal with paper). More evident form for discrete points set is numerical – they are presented by coordinates of 2D or 3D vectors.
Lines data represent different types of area boundaries, linear mapping objects, contour lines, profiles or grids. This data too can have analogue or digital presentation. In analogue form the lines are drawn on maps but in digital form the data are recorded as a sequence of points representing the line. Analogue data usual are 2D (contour lines have height but all points of the line lie in the same plane. Digital lines could be 2D or 3D. More often digitally registrated lines are 3D.
Image data are 2D continues field of values. Analogue images are photos, orthrectified images, photo plans e.t.c. Digital images are matrixes with defined size that represents initial photos, orthimages or orthomaps. In digital form the images consist of discrete data but accordingly to the theorm for digitizing resolution we may obtain practically (and prove theorticaly) the same image as in analogue form.
The problem of all plotting methods is to reconstruct the position of the object points. This reconstruction is possible if two or more photographs contain images of the same object or part of them. The directions of two or more photographs correspond roughly to the normal case

(approximately vertical aerial photographs) and the length of the base is so chosen that photographs overlapped 60%. Such stereopair could observed trough stereoscope or device for stereo observation. The observer could see stereomodel, which is sometimes called an “optical model” of the object. The registration of this model is made methods of stereoplotting.

The methods adopted for the restitution of a stereopair depend mainly on the following conditions:

- whether the elements of outer orientation of both photographs (of the stereomodel) are known or not
- whether the restitution is to be achieved numerically (starting with measured image coordinates) or with the help of an optical-mechanical instrument (analogue stereoplotter);
- whether during the restitution points are measured stereoscopically on monoscopic.

5.1.1. Restitution with known outer orientation

a) Numerical solution

For two points with 3 space coordinates there exists 4 image coordinates. This mean that there is an excess of measurements. There are four relations connecting the image and space coordinates of the point. They are based on simplified form of space similarity transformation. The according formulas are given in the appendix 5.1.

In the normal case of photogrammetry and especially for stereometric cameras with fixed base and orientation the relations are particularly simple.

$$X = x_1.\frac{B}{p_x} = x_1.\frac{B}{x_1 - x_2}$$
$$Y = c.\frac{B}{p_x} = c.\frac{B}{x_1 - x_2} \qquad (5.1)$$
$$Z = z_1.\frac{B}{p_x} = z_1.\frac{B}{x_1 - x_2}$$

b) Analogue solution

The restitution of model is made by usage of some analogue model

The principle of analogue stereoplotter is shown on the figure

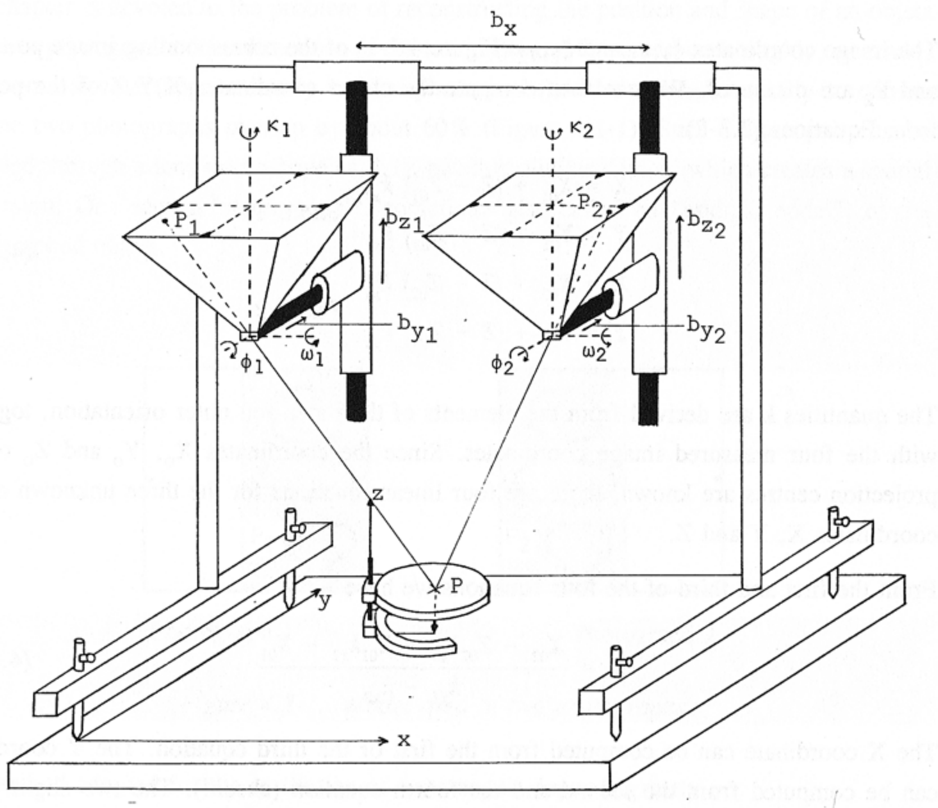

Figure 5.1. Analogue stereoplotter (optical restitution)

In analogue apparatus the coordinates of model x, y, z are:

- Reduced in scale (model scale);

- Displaced in three directions (origin of the plot sheet and zero pint of Z scale;

- Possibly rotated an angle κ (rotation of plot sheet).

5.1.2. Unknown outer orientation

The following 12 orientation elements have to be determined using control points:

Photo 1: $X_{01}, Y_{01}, Z_{01}, \omega_1, \phi_1, \kappa_1$

Photo 2: $X_{02}, Y_{02}, Z_{02}, \omega_2, \phi_2, \kappa_2$

The methods could be separated into three groups

a) Separate orientation of the two photographs

The solution requires by six equation per each point of the type:

$$\xi_i = f_x(\xi_0, c, \underline{X_0}, \underline{Y_0}, \underline{Z_0}, \underline{\omega}, \underline{\phi}, \underline{\kappa}, X_i, Y_i, Z_i)$$
$$\eta_i = f_y(\eta_0, c, \underline{X_0}, \underline{Y_0}, \underline{Z_0}, \underline{\omega}, \underline{\phi}, \underline{\kappa}, X_i, Y_i, Z_i)$$

where unknowns are underlined.

The equations can be solved after linearisation. The procedure is called spatial resection.

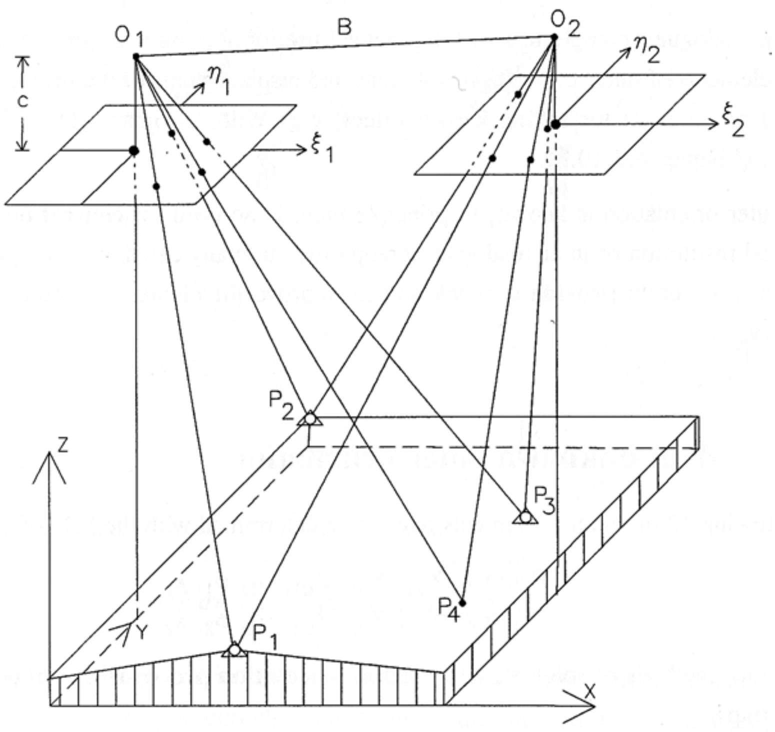

Figure 5.2. Stereomodel with three control points

The procedure has following disadvantages (by Kraus):

The information about intersection of homologues rays in same object point is not used (for extra points e.g. P_4).

Orientation requires 3 full control points (with 9 coordinates)

Procedure cannot be applied in analogue stereoplotter

b) Single step common orientation

Control points and new points are used.

The 6 equations for control points have the form.

$$\xi_{i1} = f_x(\xi_0, c, \underline{X_{01}}, \underline{Y_{01}}, \underline{Z_{01}}, \underline{\omega_1}, \underline{\phi_1}, \underline{\kappa_1}, X_i, Y_i, Z_i)$$
$$\eta_{i1} = f_y(\eta_0, c, \underline{X_{01}}, \underline{Y_{01}}, \underline{Z_{01}}, \underline{\omega_1}, \underline{\phi_1}, \underline{\kappa_1}, X_i, Y_i, Z_i)$$

Photo 1

$$\xi_{i2} = f_x(\xi_0, c, \underline{X_{02}}, \underline{Y_{02}}, \underline{Z_{02}}, \underline{\omega_2}, \underline{\phi_2}, \underline{\kappa_2}, X_i, Y_i, Z_i)$$
$$\eta_{i2} = f_y(\eta_0, c, \underline{X_{02}}, \underline{Y_{02}}, \underline{Z_{02}}, \underline{\omega_2}, \underline{\phi_2}, \underline{\kappa_2}, X_i, Y_i, Z_i)$$

Photo 2

The unknowns are underlined.

For the new points there are three further unknowns (double underlined)

$$\xi_{i1} = f_x(\xi_0, c, \underline{X_{01}}, \underline{Y_{01}}, \underline{Z_{01}}, \underline{\omega_1}, \underline{\phi_1}, \underline{\kappa_1}, \underline{\underline{X_i}}, \underline{\underline{Y_i}}, \underline{\underline{Z_i}})$$
$$\eta_{i1} = f_y(\eta_0, c, \underline{X_{01}}, \underline{Y_{01}}, \underline{Z_{01}}, \underline{\omega_1}, \underline{\phi_1}, \underline{\kappa_1}, \underline{\underline{X_i}}, \underline{\underline{Y_i}}, \underline{\underline{Z_i}})$$

Photo 1

$$\xi_{i2} = f_x(\xi_0, c, \underline{X_{02}}, \underline{Y_{02}}, \underline{Z_{02}}, \underline{\omega_2}, \underline{\phi_2}, \underline{\kappa_2}, \underline{\underline{X_i}}, \underline{\underline{Y_i}}, \underline{\underline{Z_i}})$$
$$\eta_{i2} = f_y(\eta_0, c, \underline{X_{02}}, \underline{Y_{02}}, \underline{Z_{02}}, \underline{\omega_2}, \underline{\phi_2}, \underline{\kappa_2}, \underline{\underline{X_i}}, \underline{\underline{Y_i}}, \underline{\underline{Z_i}})$$

Photo 2

The procedure of orientation is most accurate as it is used the adjustment based on direct relations between observation and unknowns.

The disadvantage of the procedure is that it could not be applied in analogue stereoplotter.

c)Two step of common orientation procedure

The orientation procedure is divided into two steps. In the first step a spatial stereomodel is created in arbitrary xyz (model) coordinate system. In the second step this model is transformed into the object XYZ coordinate system.

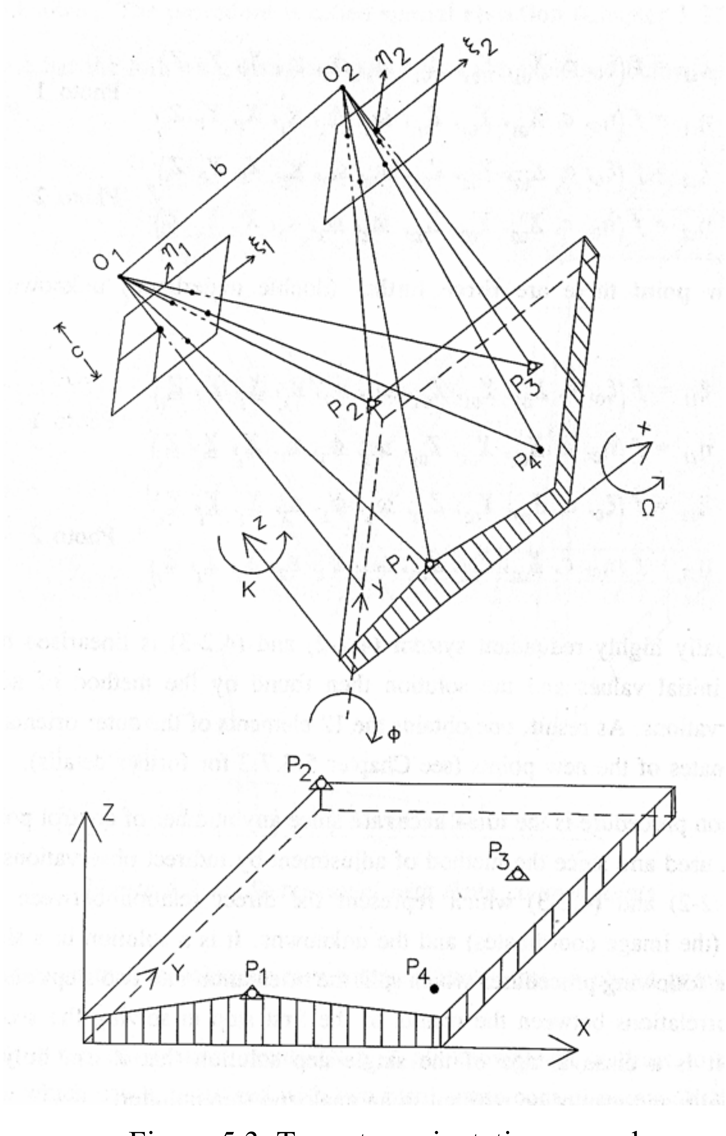

Figure 5.3. Two-step orientation procedure

First step creates model in the independent coordinate system. As the position, scale and orientation of the model are arbitrary only five parameters are required to determine this model. The choice of the parameters depends on the definition of coordinate system. The procedure is based on the intersection of homologues rays to the same object point. The number of required point is equal to the number of unknowns – five. No control points are needed for the procedure.

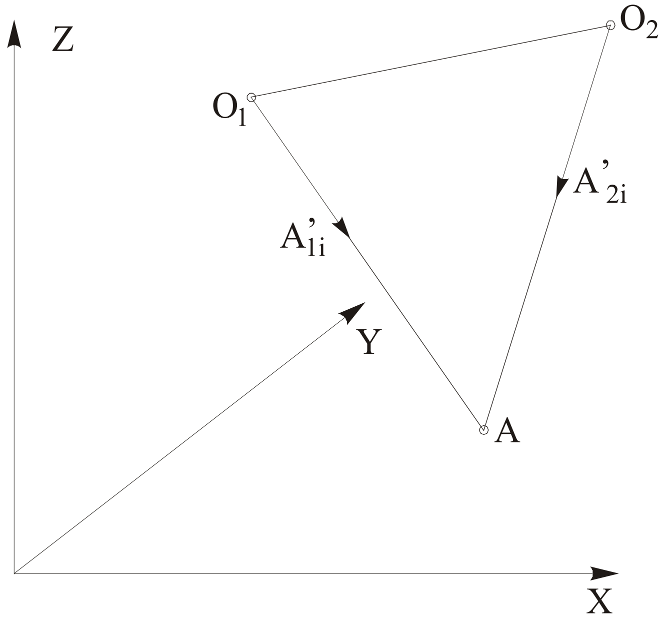

Figure 5.4. General intersection condition

The mathematical expression for coplanarity condition is the zero value of scalar triple product of the base vector and vectors to the points' images (in space).

Graphical presentation of coplanarity condition is shown in the next figure.

$$\left(b, p^{'}_{1i}, p_{'}\right) = 0 \qquad i = (1 \div 5) \tag{5.2}$$

The coplanarity equation expresses the requirement for lying of three vectors in the same plane.

Each point allows determination

The second step of procedure is based on the relation between the coordinates in model and object space.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_u \\ Y_u \\ Z_u \end{pmatrix} + m.R. \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{5.3}$$

where

Xu, Yu, Zu – object coordinates of the origin of model coordinate system xyz

m – scale number of xyz system

R – spatial rotation matrix of rotation of xyz into the XYZ system, defined by three rotation angles Φ, Ω, K.

Seven parameters defined above are called elements of the absolute orientation. Equation represents spatial similarity transformation. For finding the solution at least seven equations are necessary.

The number of equations obtained for different type of points.

- Three equations for full control point (X, Y, Z).
- Two equations for a planimetric control point (X, Y)
- One equation for height control point (Z)

To determine elements of absolute orientation it is possible to use different sets of points:

more then two planimetric points and three height points;

more then two full control points and one height control point;

The points must not lie on same straight line.

This procedure could be applied in analogue stereoplotters too.

Numerical relative orientation can be made using analytical form of coplanarity equation. For the case with small relative angles between photos (nearly vertical case or nearly normal case)

Following the simplification for rotation matrixes [Kraus K.] we obtain two possibilities for relative orientation

Relative orientation by rotations – the photographs are rotated but remain in the same position (independent srereopair)

The equation for calculation of vertical parallax is

$$p_\eta = -\xi_1 d\kappa_1 + \xi_2 d\kappa_2 + \frac{\xi_1\eta_1}{c} d\varphi_1 - \frac{\xi_2\eta_2}{c} d\varphi_2 + \left(c + \frac{\eta_2^2}{c}\right) d\omega_2 \qquad (5.4)$$

Relative orientation of successive photographs -one photograph remains fixed as it is already oriented and other is shifted and rotated (sequence of photos in strip)

$$p_\eta = -\frac{c}{h} db_y + \frac{\xi_2}{h} db_z - \frac{\xi_2\eta_2}{c} d\varphi_2 + \left(c + \frac{\eta_2^2}{c}\right) d\omega_2 + \xi_2 d\kappa_2 \qquad (5.5)$$

If only 5 points are measured it is possible to find the solution. If more points are measured the least square method must be used to find the solution.

5.1.2. Single image processing (mono plotting)

Main advantage in photogrammetry is the possibility to restitute the space information from two dimensional images – that's means stereo photogrammetry. There are some technologies where the single photo usage is possible. In this case sometimes is possible to draw sketches without tacking in account the perspective deformation and distance to the objects. There are possible cases when only perspective deformations are considered. There are no instruments for such type of photogrammetric plotting. But some methods of orthphoto transformation are based on this principle.

Other possibility is the usage of known model of the object. For topographic mapping this is the information for terrain in the form of Digital Elevation Model (DEM). The possible used models are triangle and grid model. For purposes of photogrammetry the application of regular model is most suitable. In case of close range photogrammetry this approach is based of information about the object – building, car or other engineering construction. It is in the form of 3D skeleton model.

Other possibility is usage of structural light. This is most often projecting dark and white strips over the object. In such way it is possible to distinguish the shape and size of the object and to measure the parameters.

5.2. Application and products of different methods

5.2.1.Stereo methods

The utilization of stereo methods is for the purposes of:

- point coordinates restitution;

Points can be determinate by analytical methods and present in paper or digital

- line drawing;

Line drawings could be in analogue form in drawing maps or in the digital model as the set of points with their description depending on method of line codding.

- collection of set of points

This set can be regular or irregular, depending on the type of instrument and method of measuring

Table 5.1

Type of data	Result	Area of application	Data format of product	Method of processing	Apparatus
Discrete points	Point position	Aerial triangulation	coordinates on paper forms or computer files	analytical space resection	stereo comparator analytical stereoplotter

			sign in the map	analogue restituion	analogue stereoplotter
Line	Presentation of lines	Topographic mapping Architectural mapping	set of line points with line description in some graphic system	measuring and processing of discrete points over the lines	stereo comparator analytical stereoplotter digital system
			line drawing map	continuous drawing	analogue stereoplotter
Point set	Regular or irregular grid	Terrain models Object archiving	set of points with description of triangles matrix of regular grid points	manual or automatic measuring of the grid points	stereo comparator analytical stereoplotter digital system
	regular grid		signs on the map with points' heights	point registration	analogue stereoplotter

5.2.2. Products of mono plotting

Main mono plotting application is image rectification. It can be used with information about the object and without information about the object

Other mono plotting application is reconstruction of perspective image of the plane objects and reconstruction of the surface illuminated with structural light. Main area of application is engineering (industrial) photogrammetry.

5.3. Difference between stereoscopic and point wise plotting

Stereoscopic measurement requires more sophisticated methods and complicated apparatus. But the accuracy and reliability of point measuring is higher. The first reason is that this is the natural way of object observing. Second is that in monoscopic measuring is not taken into account the structure of the object and is possible not to measure the same point. The results from monoscopic and stereoscopic measurement may be the same only in the cases of planar objects, that lye on the plat surface. The idea of this is shown on the next figure.

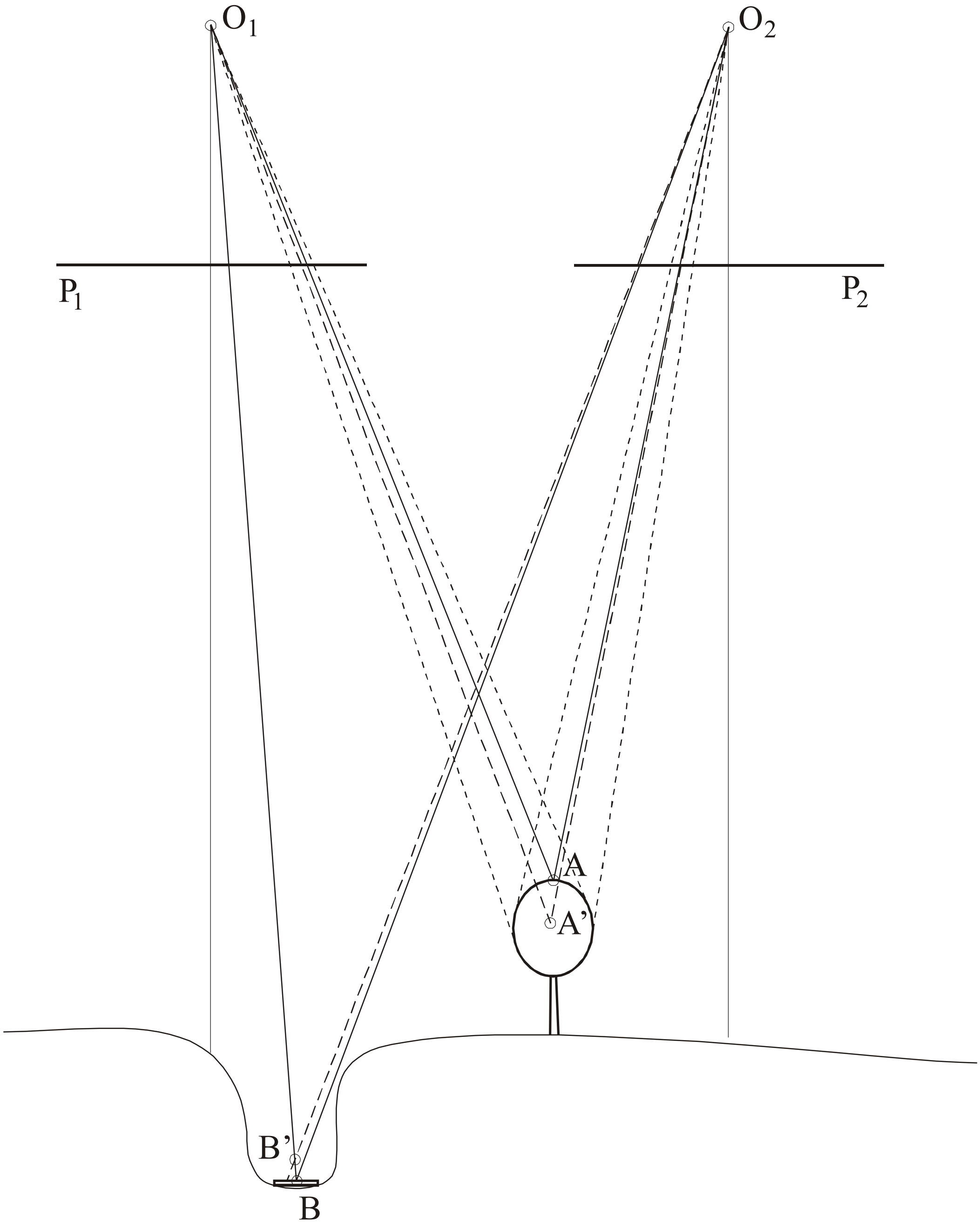

Figure 5.5. Errors between stereoscopic and monoscopic measurements

There are some tasks and cases in close range photogrammetry when it is not possible to apply stereo scoping measurment because the requirements of stereoscopic observation are not satisfied, because the angles between main rays of images are very high..

5.3.2. Monopllotting and stereoplotting in digital images

Automatic measurement of fiducial marks and tie points is based on correlation method. The cross -correlation function is calculated and from it the the parameters (coordinates of points are obtained. The derivation of correlation function is given in the Appendix 5.2.

$$D_3(m,n) = \sum_j \sum_k [T(j-m, k-n)]^2$$

The energy of pattern is fixed. Most important is function of cross-correlation. Usually it is used for measurement of the position of the object. When it is used the result is compared with some threshold value $L_R(m,n)$.

$$R_{FT}(m,n) > L_R(m,n)$$

Measuring of fiducial marks

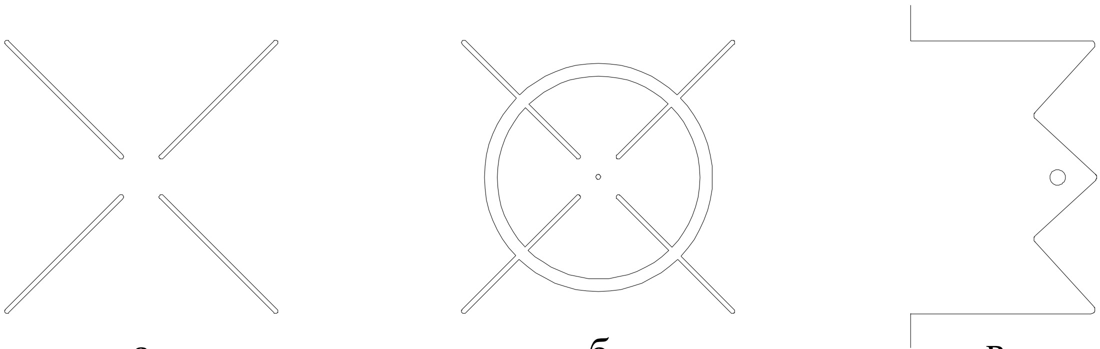

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Figure 5.7. Vector patterns of different types of fiducial marks

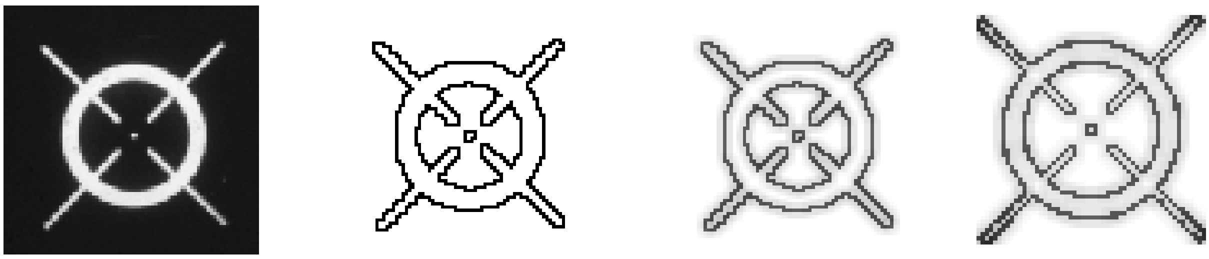

Figure 5.8. Fiducial mark grey level image — Contour image of mark formed from image — Grey level mark model from contour image — Grey level model from vector pattern

In similar way is possible to find the position of control points marked on the terrain, if their shape is fixed and is possible to create the corresponding pattern.

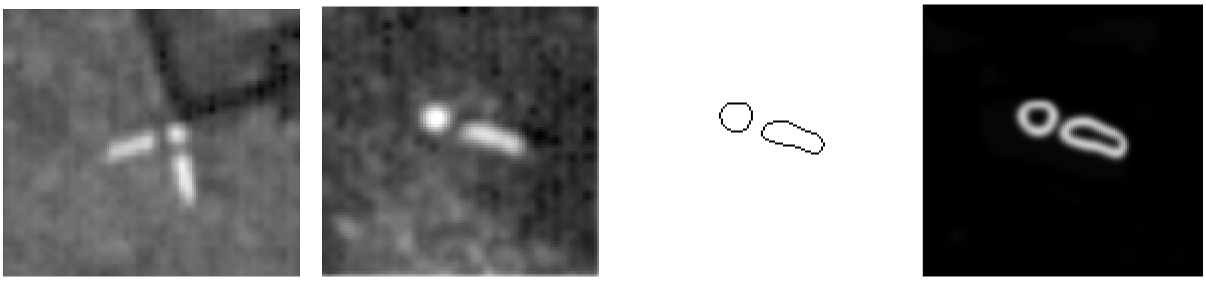

Figure 5.9. Two arrows mark grey image | One arrow mark grey image | Vector pattern of the mark | Contour image formed from image

Appendixes

Appendix 5.1.

Numerical solution for space coordinates for the case of parallel coordinate systems

$$\begin{aligned} X &= X_{01} + (Z - Z_{01}).k_{x1} \quad (1) \\ Y &= Y_{01} + (Z - Z_{01}).k_{y1} \quad (2) \\ X &= X_{02} + (Z - Z_{02}).k_{x2} \quad (3) \\ Y &= Y_{02} + (Z - Z_{02}).k_{y2} \quad (4) \end{aligned} \tag{5.6}$$

The value of Z could be obtained by equations (1) and (3)

$$Z = \frac{X_{02} - Z_{02}.k_{x2} + Z_{01}.k_{x1} - X_{01}}{k_{x1} - k_{x2}} \tag{5.7}$$

As the value of Z is determined from (1) and (3) the usage of any of these equations gives the same result. The Y coordinate can be computed by (2) or (4) and if the values of image coordinates are not adjusted the mean value of two different values must be calculated.

Appendix 5.2.

Different measures of difference between the objects

$$D_N(m,n) = \sum_j \sum_k |F(j,k) - T(j-m,k-n)| \tag{5.8}$$

Absolute value

$$D_E(m,n) = \sqrt{\sum_j \sum_k F(j,k) - T(j-m,k-n)]^2} \tag{5.9}$$

Euclidian distance

$$D_M(m,n) = \max_{j,k} |F(j,k) - T(j-m,k-n)| \tag{5.10}$$

Maximum absolute difference

Correlation methods for processing

Main importance has similarity function based on Euclidian metris. After expansion of the expression the result is:

$$D_E(m,n) = D_1(m,n) - 2.D_2(m,n) + D_3(m,n) \quad (5.11)$$

where

$$D_1(m,n) = \sum_j \sum_k [F(j,k)]^2 \quad (5.12)$$

corresponds to the energy of image in the correlation window,

$$D_2(m,n) = R_{FT}(m,n) = \sum_j \sum_k [F(j,k).T(j-m,k-n)] \quad (5.13)$$

is cross-correlation function between object and pattern and

$$D_3(m,n) = \sum_j \sum_k [T(j-m,k-n)]^2 \quad (5.14)$$

is the energy of pattern.

The energy of pattern is fixed. The most important is function of cross-correlation.